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Shell-Crossing and Shell-Focusing Singularity in Spherically Symmetric Spacetime

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ABSTRACT

Starting from Schwarzschild exterior solution singularity has attracted researchers. There are conjectures (like cosmic censorship conjecture) which suggest that the singularities are to be covered by event horizon. However these are examples of naked singularities, thus it is always interesting to discuss singularities in this context.

Spacetime singularities are classified as shell-crossing and shell-focusing also; in this paper we consider spherically symmetric spacetimes filled with dust and perfect fluid. We find that in these cases both type singularities occur simultaneously.

Keywords: Shell-crossing, Shell-focusing, Naked singularity

INTRODUCTION

The Einstein's theory of relativity prophecies that the end fate of collapsing matter which exhausted its threadbare nuclear fuel. If star is more massive than few solar masses it could undergo an endless gravitational collapse without earning any stable stage. When the star wiped out it's inter nuclear fuel which provided outward pressure against inward pull of gravitation filed

The key point of this analysis is to find out the behavior of spacetime singularity of collapsing matter cloud in which the radius of star converse to a small positive value. The result form gravitational collapse is highly dense region with strong gravity and physical quantity such as density and Kretschmann curvature scalar could blow-up. If the event horizon starts developing before such a collapse, the collapsing matter gets hidden within a horizon. As result of collapse black hole created. If the developments of event horizon gets late during the collapse then the final result is the naked singularity, which can send out information to observer from strong gravity region. The cosmic censorship conjecture showed that trapped surface gives spacetime singularity; such a singularity must always hide behind event horizon of gravity.

The gravitational collapse of massive cloud could it be inspected using Einstein field equations. Here the main point in the theory of collapsing matter cloud is that the creation of shell-crossing and shell-focusing singularities. The shell of matter implode in such a way that fast moving outer shell of matter overtake the inner shells and producing a weak singularity this is called shell-crossing singularity where density and curvature scalar $K = R^{hijk}R_{hijk}$ blow-up.

On the other hand shell-focusing singularity occurring at center of spherically symmetric collapsing matter, this is genuine curvature singularity where Kretschmann curvature scalar assume unboundedly large value. Many times singularity is observed due to bad choice of coordinate. To examine a singularity due to coordinates is done by checking the Kretschmann curvature scalar. We consider shell-crossing as a weak singularity because the volume element along to geodesics are non-zero, we can remove shell-crossing singularity with suitable extension of spacetime.

Spherically Symmetric Spacetime

The final stage of gravitational collapsing matter we consider here idealization, spherically symmetry. The advantage is that it can solve analytically to get exact solution. The first studies examining the dynamical collapsing matter cloud by Oppenheimer and Snyder [3], and Datt [9]. The naked nature of gravitational collapse of inhomogeneous dust cloud (zero pressure) has been studies in details by Datt [9]. Here we need investigating only the gravitational collapse with two real orthogonal eigenvectors of spherically symmetric metric. We take the matter field with weak energy condition. The energy density as measured by any observer is positive for any timelike vector V^i.

$$T_{ij}V^iV^j \geq 0 \qquad (1)$$

Shell-Crossing and Shell-Focusing Singularity

Here we consider perfect matter fluid, the idealized condition and in order to determine field equation and analyze shell-crossing and shell-focusing singularity. Such matter field, can be given as

$$T_{ij} = (\rho + p)V_iV_j - pg_{ij} \qquad (2)$$

For a spherically symmetric matter distribution, choose the co-ordinates t, r, θ, ϕ, the metric is written as,

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}dr^2 + R^2 d\Omega^2, \qquad (3)$$

where $d\Omega^2 = d\theta^2 + sin\theta\, d\phi^2$ and ν, ψ and R are function of t and r and the stress energy tensor T^{ij} as given by equation (2) has only diagonal component in co-moving coordinate system

$$T_{tt} = -\rho\, , T_{rr} = T_{\theta\theta} = T_{\phi\phi} = p\, , \qquad (4)$$
$$T_{ij} = 0, \qquad for\ \ i \neq j$$

Here ρ is density and p is pressure which are eigenvalues of T^{ij}. Here the energy density as measured by any observer must be positive. The weak energy condition holds for matter with two eigenvectors provided,

$$\rho \geq 0, \rho + p \geq 0 \qquad (5)$$

The Einstein field equations for metric (3) gives,

$$p = \frac{1}{R^2}\left[\begin{matrix} e^{-2\nu}\left(-(\dot{\nu} - \dot{\psi})(\dot{R} - R\dot{\psi}) + \ddot{R} + R\ddot{\psi}\right) \\ -e^{-2\psi}\left((R' + R\nu')(\nu' - \psi') + R + \mathrm{R}\nu\right) \end{matrix}\right] (6)$$

$$= \frac{1}{R^2}\left[\begin{matrix} -e^{-2\psi}R'(R' + 2R\nu') + e^{-2\nu}(e^{2\nu} + \dot{R}^2 \\ -2R\dot{R}\nu + 2\ddot{R}) \end{matrix}\right] \quad (7)$$

$$\rho = \frac{1}{R^2}[e^{-2\psi}(R'^2 - 2RR'\psi' + 2RR'')$$
$$+e^{-2\nu}(e^{-2\nu} + \dot{R}^2 - 2R\dot{R}\dot{\psi})] \qquad (8)$$

$$0 = -2[\psi R' + R\nu' - R'] \qquad (9)$$

Now,

$$\rho = \frac{1}{R^2}[e^{-2\psi}(R'^2 - 2RR'\psi' + 2RR'')$$
$$+e^{-2\nu}(e^{-2\nu} + \dot{R}^2 - 2R\dot{R}\dot{\psi})]$$

Using, $-2[\dot{\psi}R' + \dot{R}\nu' - \dot{R}'] = 0$,

$$\rho = \frac{\begin{matrix} e^{-\psi}(R'^2 + 2RR'\psi' - RR') \\ +e^{-2\nu}\left(\dot{R}^2 + \frac{2}{R'}(-R\dot{R}\nu' + R\dot{R}\dot{R}')\right) + 1 \end{matrix}}{R^2} \qquad (10)$$

Or,

$$\rho = \frac{1}{R^2R'}\left[\begin{matrix} R'\{1 + e^{-2\nu}\dot{R}^2 - e^{-2\psi}R'^2\} \\ +R\{2e^{-2\nu}\dot{R}(R'\dot{R}'R" - \dot{R}\nu') + 2e^{-2\psi}R'^2\psi'\} \end{matrix}\right] \qquad (11)$$

$$\rho = \frac{-F'}{R^2R'} \qquad (12)$$

$$p = \frac{\dot{F}}{R^2\dot{R}} \qquad (13)$$

$$2R' = 2R'\frac{\dot{G}}{G} + \dot{R}\frac{H'}{H} \qquad (14)$$

$$\nu' = \frac{p'}{\rho + p} \qquad (15)$$

$$F = R(1 - G + H) \qquad (16)$$

Functions H and G are defined as,

$$H = e^{-2\nu(r,t)}\dot{R}^2$$

$$G = e^{-2\psi(r,t)}R'^2$$

Here the prime and dot denote partial derivative with respect to the parameters r and t respectively. $F = F(r,t)$ the Misner-Sharp mass function with $F \geq 0$. In above non-linear system we have five equation and six unknowns that is to say R, F, G, H and ρ and p. Here ρ is the positive density; p is pressure which is uniform throughout collapse. We are only affiliated with the gravitational collapse of perfect fluid. For the collapse condition,

$$\dot{R}(t,r) < 0$$

In general, R' may not be positive. The inner shells are more weak then outer shells.
For spherically symmetric continual collapse energy-mass density is given by.

$$\rho = \frac{F'}{R^2R'}$$

This condition trivially leads to shell-crossing and shell-focusing, generally this shell-crossing as weak singularity with dust ball and perfect fluid. The volume elements along the geodesics are converse to non-zero so this is weak surface singularity.
The shell-crossing singularities were studied by Szekeres [4] and Lun [4] who investigated Newtonian spherically symmetric dust ball solution. In Tolman dust ball solution Hellaby [2] and lake [2](1995) define necessary and sufficient condition for shell-crossing.

CONCLUSION

The study of spherically symmetric collapse out here reveals that non-removable shell-crossing singularity occurs. Also, the singularity here is genuine shell-focusing curvature singularity. Here we considered perfect fluid as material content of spherical body. Our results match with the dust situation considered in Datt [9]. We hope that this method can be applied in more general situation.

REFERENCES

[1] A. Meszaros, On Shell-Crossing in the Tolman Metric, Mon. Not. R. Astr. Soc. Vol. 253, No. 619 (1991)

[2] C. Hellaby and K. Lake, Astrophys. J.Vol.290 No. 381(1985), Erratum Vol.300 No.461 (1986)

[3] J. R. Oppenheimer and H. Snyder, Phys. Rev. Vol.56, No. 455 (1939)

[4] P. Szekeres and A. Lun, J. Austral. Math. Soc. Ser. B. Vol.41 No.167 (1999)

[5] P. S. Joshi and I. H. Dwivedi, Phys. Rev. D Vol. 47 No.5357 (1993)

[6] P. S. Joshi, Gravitational Collapse and Spacetime Singularities Cambridge University Press, Cambridge (2007).

[7] P. S. Joshi, Global Aspects in Gravitational and cosmology, Oxford Uni. press Inc., New York (1993).

[8] P. S. Joshi, Shell-Crossing in gravitational Collapse, Int. Jour. of Mod. Phy., Vol. 22, No.5 (2013)

[9] S. Datt, Z. Phys. Vol.108, No. 314(1938)

[10] S. W. Hawking and G. F. R. Ellis, The large scale structure of spacetime, Cambridge Uni. Press, Cambridge. (1973)