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EFFECT OF EXOTIC PREDATOR ON A NATIVE PREY-PREDATOR SYSTEM WITH SPECIAL REFERENCE TO KUNO WILDLIFE SANCTUARY IN INDIA: A MODEL

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ABSTRACT

In this paper, a mathematical model is proposed to study the effect of exotic lion population on prey-predator system consisting of native herbivore population and native leopard population with special reference to Kuno Wildlife Sanctuary in India. The model includes three state variables viz; density of herbivore population, density of leopard population and density of exotic lion population. Stability analysis of all the feasible equilibrium points of the models is carried out. We concluded that interior equilibrium point becomes more stable if inter specific interference coefficient due to exotic lion species as well as constant recruitment rate of lion population decreased. It is also pointed out that the recruitment rate of Asiatic lion plays as a critical role in the dynamical behaviour of the system. Finally numerical simulation is carried out to support the analytical results of the models.

KEYWORDS:- Native species; Exotic species; Equilibia; Biological invasion; Stability

INTRODUCTION

The dynamic relationship between predators and their prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance [1-7]. Many authors have studied effect of exotic predator on native prey species using mathematical models [8-10]. The interesting study of realistic mathematical models in theoretical ecology is a very good reflection of their use in helping to understand the dynamic process involved and in making practical prediction.

It has been noticed in the Gir National Park located in Gujrat in India that the population of lions and have attained an equilibrium level and the expansion limits have been reached. There are large scale deaths in the lion population annually because of overcrowding increasing and intraspecific competition. Asiatic lion prides require large territories but there is limited space at Gir wildlife sanctuary, which is boxed in on all sides by heavy human habitation.

Kuno Wildlife Sanctuary or Palpur-Kuno Wildlife Sanctuary (between latitudes of 25°30’-25°53’N & longitude of 77°07’-77°26’E) lies in the Sheopur district of north western Madhya Pradesh, a state in central India [11]. An area of 344.686 square kilometers was set aside as a Wildlife Sanctuary in 1981. This park is home to many species of wild animals including wolves, monkeys, leopards, nilgai and possibly a few remaining Bengal Tigers.

Government of India planned to shift the Asiatic lion from Gir National Park to other places. The Kuno Wildlife Sanctuary was selected as the reintroduction site for the endangered Asiatic lion because it is the former range of the lions before it was hunted into extinction in about 1873. Currently the Asiatic lion reintroduction project is underway. The lions are to be reintroduced from Gir Wildlife Sanctuary in the neighboring Indian state of Gujarat where they are currently overpopulated. The reintroduction project of Asiatic lion to other wildlife sanctuary such as Kuno National Park, requires quantitative investigation to predict the future scenario of the Kuno National Park with respect to survival or extinction of both native and exotic populations.

In view of this the main purpose of this paper is to construct a model to study the effect of exotic species (Asiatic lion) on the growth dynamics of native prey (herbivore) and native predator leopard species.

BASIC ASSUMPTIONS AND MATHEMATICAL MODEL

Let $x(t)$ denotes the density of herbivore population, $y(t)$ denotes the density of leopard population and $z(t)$ denotes the density of exotic lion (Asiatic) population. It is assumed that at present the Asiatic lion is exotic to the habitat

(Kuno National Park). It is also assumed that the herbivore population has a logistic growth rate, linear interaction with leopard population on account of exotic lion interference and type of interaction considered in the model with exotic lion population is taken from [12-13]. It is a well-known fact that a lion does not kill the prey if it is not hungry. It will primarily kill feed, therefore, the predation term in the equation describing lion dynamics will depend on the prey and predator densities, as well as Type II functional response. We assume that r and k are the growth rate and the carrying capacity of herbivore population respectively. d_1 is death rate of leopard population. d_2 is the death rate of exotic lion population. m is the constant recruitment rate of Asiatic lion (exotic) under the reintroduction project. a is the predation rate of herbivore population by lion population. b is the predation rate of herbivore population by leopard population. e_1 and e_2 are conversion efficiencies of native leopard species and exotic lion species. f is the decay rate of leopard species due to predation by lion species. c is the interspecific interference coefficient due to exotic lion species. h is the intraspecific interference coefficient due to exotic lion species, which is determined by the population size of the exotic lion. In view of the above, the resultant system dynamics is governed by the following system of differential equations:

Model 1 (With exotic species)

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - axz - \frac{bxy}{1+cz}, \qquad (2.1)$$

$$\frac{dy}{dt} = \frac{be_1xy}{1+cz} - d_1y - fyz, \qquad (2.2)$$

$$\frac{dz}{dt} = m + \frac{ae_2xz}{1+hz} - d_2z, \qquad (2.3)$$

with initial condition as $x(0) \geq 0, y(0) \geq 0$ and $z(0) \geq 0$.

Where $r, k, a, b, c, f, h, m, d_1, d_2, e_1, e_2$ are positive constants.

In the absence of exotic lion species the above system (2.1)-(2.3) is governed by the following system of differential equations:

Model 2 (Without exotic species)

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - bxy, \qquad (2.4)$$

$$\frac{dy}{dt} = be_1xy - d_1y, \qquad (2.5)$$

with non-negative initial condition as $x(0) \geq 0$ and $y(0) \geq 0$.

EQUILLIBRIA OF THE MODELS 1 AND 2

In this section, we analyze the system of equations (2.4)-(2.5) under the initial conditions. We find all the possible feasible equillibria of the system of equations (2.4)-(2.5). The system has three feasible equillibria, namely

(i) Trivial equilibrium point $E_T \equiv (0,0)$.

(ii) Axial equilibrium point $E_A \equiv (k, 0)$.

(iii) Interior Equilibrium point $E_P \equiv (x^\#, y^\#)$.

Where,

$$x^\# = \frac{d_1}{be_1}, y^\# = \frac{r(be_1k - d_1)}{b^2e_1k}.$$

Equilibrium point E_P exists if $be_1k > d_1$.

We now analyze the system of equations (2.1)-(2.3) under the initial conditions. We find all the possible feasible equillibria of the system of equations (2.1)-(2.3). The system has two feasible equillibria, namely

(i) Boundary equilibrium point $E_{B_e} \equiv (\tilde{x}, 0, \tilde{z})$.

Where,

$$\tilde{x} = \frac{(1+h\tilde{z})(d_2\tilde{z} - m)}{ae_2\tilde{z}},$$

$$\tilde{z} = \frac{r(\beta_1 + mh) + \sqrt{r^2(\beta_1 + mh)^2 + 4mr\beta_2}}{2\beta_2},$$

$$\beta_1 = ae_2k - d_2, \beta_2 = a^2e_2k + hrd_2.$$

Equilibrium point E_{B_e} exist if $d_2\tilde{z} > m$ i.e. $m < m_1^*$.

Where,

$$m_1^* = \frac{rd_2}{a}$$

(ii) Interior Equilibrium point $E_{P_e} \equiv (x^*, y^*, z^*)$.

Where,

$$z^* = \frac{\beta_3x^* - \beta_4}{\beta_5 + \beta_6x^*} = g(x^*),$$

$$y^* = \frac{[r\beta_7 - \beta_8g(x^*) - \beta_9g^2(x^*)]\left(1 + cg(x^*)\right)}{b^2e_1k},$$

$\beta_3 = bhd_2e_1, \beta_4 = hd_1d_2 + cfm,$

$\beta_5 = chd_1d_2 + cfhm + f(h-c)d_2$

$\beta_6 = acfe_2, \beta_7 = be_1k - d_1,$

$\beta_8 = fr + cd_1r + abe_1k, \beta_9 = cfr$

x^* is the root of the equation

$$G_1x^{*3} + G_2x^{*2} + G_3x^* + G_4 = 0.$$

Where,
$G_1 = be_1\beta_6^2,$
$G_2 = 2be_1\beta_5\beta_6 - d_1\beta_6^2 - \beta_3\beta_6\beta_{10} - cf\beta_3^2,$
$G_3 = be_1\beta_5^2 - 2d_1\beta_5\beta_6 - \beta_3\beta_5\beta_{10} + \beta_4\beta_6\beta_{10} + 2cf\beta_3\beta_4,$
$G_4 = \beta_4\beta_5\beta_{10} - cf\beta_4^2 - d_1\beta_5^2, \beta_{10} = f + cd_1.$
Hence unique root x^* is positive if $G_2 > 0$ and $G_4 < 0$.
Therefore equilibrium point $E_{P_e} \equiv (x^*, y^*, z^*)$ exists if
$\beta_3 x^* > \beta_4, h > c, r\beta_7 > \beta_8 g(x^*) + \beta_9 g^2(x^*),$
$2be_1\beta_5\beta_6 > d_1\beta_6^2 + \beta_3\beta_6\beta_{10} + cf\beta_3^2,$
$cf\beta_4^2 + d_1\beta_5^2 > \beta_4\beta_5\beta_{10}.$

BOUNDEDNESS OF THE SOLUTIONS OF THE MODELS 1 AND 2

Lemma 4.1 All the solutions of system of equations (2.4)-(2.5) with the positive initial condition are uniformly bounded within the region $\mathbf{\Omega_1}$.
Where,

$$\mathbf{\Omega_1} = \left\{(x,y) : 0 \le x \le k, 0 \le e_1 x + y \le \frac{2e_1 kr}{\theta}, \theta = min(r, d_1)\right\}$$

for any $\theta > 0$ is a region of attraction.

Lemma 4.2 All the solutions of system (2.1)-(2.3) with the positive initial condition are uniformly bounded within the region $\mathbf{\Omega_2}$.
Where,

$$\mathbf{\Omega_2} = \left\{(x,y,z) : 0 \le x \le k, 0 \le e_1 x + y \le \frac{2e_1 kr}{\theta}, 0 \le z \le \frac{m}{d_2 - ake_2}, \theta = min(r, d_1), d_2 > ake_2\right\}$$

for any $\theta > 0$ is a region of attraction.

DYNAMICAL BEHAVIOUR OF THE MODELS 1 AND 2

In the previous section, we observed that the system of equations (2.4)-(2.5) have three feasible equilibria E_T, E_A and E_P. We will now study the dynamical behaviour of the system about all the three feasible equilibria.

(i) The variational matrix for the system of equations (2.4)-(2.5) evaluated at E_T is

$$J_{E_T} \equiv J(0,0) = \begin{bmatrix} r & 0 \\ 0 & -d_1 \end{bmatrix}.$$

The eigen values of the characteristic equation of J_{E_T} are $\lambda_1 = r$ and $\lambda_2 = -d_1$. It is seen from these eigen values that equilibrium point E_T is unstable.

(ii) The variational matrix for the system of equations (2.4)-(2.5) evaluated at E_A is

$$J_{E_A} \equiv J(k,0) = \begin{bmatrix} -r & -bk \\ 0 & bke_1 - d_1 \end{bmatrix}.$$

The eigen values of the characteristic equation of J_{E_A} are $\lambda_1 = -r$ and $\lambda_2 = bke_1 - d_1$. It is seen from these eigen values that equilibrium point E_A is stable provided $bke_1 < d_1$.

Remark 5.1 :- It may be observed from the stability condition of E_A that herbivore species will survive and leopard species will tend to extinction if the product of predation rate of leopard population due to predation of herbivore population by leopard population, conversion efficiency of leopard species and carrying capacity of herbivore population is less than death rate of leopard species.

(iii) The variational matrix for the system of equations (2.4)-(2.5) evaluated at E_P is

$$J_{E_P} \equiv J(x^\#, y^\#) = \begin{bmatrix} \frac{-rx^\#}{k} & -bx^\# \\ be_1 y^\# & 0 \end{bmatrix}.$$

The characteristic equation for variational matrix J_{E_P} is given by

$$\lambda^2 + B_1\lambda + B_2 = 0. \qquad (5.1)$$

Where,

$$B_1 = \frac{rx^\#}{k}, B_2 = b^2 e_1 x^\# y^\#.$$

Since $B_1 > 0$ and $B_2 > 0$. Therefore using Routh-Hurwitz criteria all the roots of equation (5.1) are negative or have negative real parts. Hence equilibrium point E_P is locally asymptotically stable.

Remark 5.2:- It may be observed from the stability condition of E_P that herbivore species and leopard species will survive if the product of predation rate of leopard population due to predation of herbivore population by leopard population, conversion efficiency of leopard species and carrying capacity of herbivore population is greater than death rate of leopard species. We also concluded that if equilibrium point E_P exists then equilibrium point E_A is unstable.

In the previous section, we observed that the system of equations (2.1)-(2.3) have two feasible equilibria E_{B_e} and E_{P_e} . We will now study the dynamical behaviour of the system about all the three feasible equilibria.

(i) The variational matrix for the system of equations (2.1)-(2.3) evaluated at E_{B_e} is

$$J_{E_{B_e}} \equiv J(\tilde{x},0,\tilde{z})$$
$$= \begin{bmatrix} \frac{-r\tilde{x}}{k} & \frac{-b\tilde{x}}{1+c\tilde{z}} & -a\tilde{x} \\ 0 & \frac{be_1\tilde{x}}{1+c\tilde{z}} - d_1 - f\tilde{z} & 0 \\ \frac{ae_2\tilde{z}}{1+h\tilde{z}} & 0 & -\frac{m}{\tilde{z}} - \frac{ahe_2\tilde{x}\tilde{z}}{(1+h\tilde{z})^2} \end{bmatrix}.$$

The one eigen value of characteristic equation of $J_{E_{B_e}}$ is

$$\lambda_1 = \frac{be_1\tilde{x}}{1+c\tilde{z}} - d_1 - f\tilde{z}\,.$$

The rest eigen values of characteristic equation of $J_{E_{B_e}}$ will obtain from following equation

$$\lambda^2 + C_1\lambda + C_2 = 0. \qquad (5.2)$$

Where,

$$C_1 = \frac{r\tilde{x}}{k} + \frac{m}{\tilde{z}} + \frac{ahe_2\tilde{x}\tilde{z}}{(1+h\tilde{z})^2},$$
$$C_2 = \frac{r\tilde{x}}{k}\left(\frac{m}{\tilde{z}} + \frac{ahe_2\tilde{x}\tilde{z}}{(1+h\tilde{z})^2}\right) + \frac{a^2e_2\tilde{x}\tilde{z}}{1+h\tilde{z}}.$$

Since $C_1 > 0$ and $C_2 > 0$. Therefore using Routh-Hurwitz criteria all the roots of equation (5.2) are negative or have negative real parts. Hence equilibrium point E_{B_e} is locally asymptotically stable provided

$$d_1 + f\tilde{z} > \frac{be_1\tilde{x}}{1+c\tilde{z}}.$$

Again on simple calculation, we observed that E_{B_e} is locally asymptotically stable provided $m_2^* < m$.

Where,

$$m_2^* = \frac{-H_2 + \sqrt{H_2^2 - 4H_1H_3}}{2H_1},$$
$$H_1 = \beta_9 r^2(h\beta_8 - \beta_9 + rh^2\beta_7),$$
$$H_2 = r\beta_8(\beta_2\beta_8 + r\beta_1\beta_9) + r^2\beta_7(h\beta_2\beta_8 + 2\beta_2\beta_9 + 2hr\beta_1\beta_9),$$
$$H_3 = r\beta_7(r\beta_1\beta_2\beta_8 + r^2\beta_1^2\beta_9 - r\beta_2^2\beta_7).$$

Remark 5.3:- It may be observed from the stability condition of E_{B_e} that herbivore species and exotic lion species will survive and leopard species will tend to extinction if constant recruitment rate of exotic lion population exists between two critical values m_1^* and m_2^*.

(ii) The variational matrix for the system of equations (2.1)-(2.3) evaluated at E_{P_e} is

$$J_{E_{P_e}} \equiv J(x^*,y^*,z^*)$$
$$= \begin{bmatrix} \frac{-rx^*}{k} & \frac{-bx^*}{1+cz^*} & -ax^* + \frac{bcx^*y^*}{(1+cz^*)^2} \\ \frac{be_1y^*}{1+cz^*} & 0 & -fy^* - \frac{bce_1x^*y^*}{(1+cz^*)^2} \\ \frac{ae_2z^*}{1+hz^*} & 0 & -\frac{m}{z^*} - \frac{ahe_2x^*z^*}{(1+hz^*)^2} \end{bmatrix}.$$

The characteristic equation for variational matrix $J_{E_{P_e}}$ is given by

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0. \qquad (5.3)$$

Where,

$$A_1 = \frac{rx^*}{k} + \frac{m}{z^*} + \frac{ahe_2x^*z^*}{(1+hz^*)^2},$$
$$A_2 = \frac{rx^*}{k}\left(\frac{m}{z^*} + \frac{ahe_2x^*z^*}{(1+hz^*)^2}\right) + \frac{a^2e_2x^*z^*}{1+hz^*} + \frac{bx^*y^*}{(1+cz^*)^2}\left(be_1 - \frac{ace_2z^*}{1+hz^*}\right),$$
$$A_3 = y^*\left(\frac{b^2e_1mx^*}{z^*(1+cz^*)^2} - \frac{afe_2z^*}{1+hz^*}\right) + \frac{abe_1e_2x^*y^*z^*}{(1+hz^*)(1+cz^*)^2}\left(\frac{bhx^*}{(1+hz^*)} - c\right).$$

Again on simple calculation, we observed that $A_1 > 0, A_2 > 0, A_3 > 0$ and $A_1A_2 - A_3 > 0$ if (i) $be_1 > ae_2$ (ii) $bm > ae_2z^*$ (iii) $hd_1 > ce_1$ (iv) $h > c$ are being satisfied.

Using the Routh-Hurwitz criteria, we derive that the equilibrium point E_{P_e} is locally asymptotically stable, if (i) $be_1 > ae_2$ (ii) $bm > ae_2z^*$ (iii) $hd_1 > ce_1$ (iv) $h > c$ are being satisfied.

Remark 5.4:- It may be observed from the stability condition of E_{P_e} that all the three species that is herbivore species, leopard species and exotic lion species would coexist if (i) ratio of predation rates of leopard to lion is greater than the ratio of conversion efficiencies of lion to leopard, (ii) product of constant recruitment rate of exotic lion population and predation rate of herbivore population by leopard population is greater than product of predation rate of herbivore population by exotic lion population, conversion efficiency and equilibrium level of exotic lion population, (iii) ratio of intra specific interference coefficient within exotic lion species to inter specific interference coefficient due to exotic lion species is greater than ratio of conversion efficiency of leopard population to death rate of leopard population, and (iv) the intra specific interference coefficient within exotic lion species is greater than the inter specific interference coefficient due to exotic lion species.

NUMERICAL EXAMPLE

In this section, we present a simulation analysis to explain the applicability of results in model 1 and model 2. Now we choose the following values of parameters in model 2

$r = 1.5; k = 100;; b = 0.15; e_1 = 0.1; d_1 = 0.5$. Using above set of parameters, the stability region Ω_1 and interior equilibrium point E_P is given by

$$\Omega_1 = \{(x, y) \in R_+^2 : 0 \le x \le 100, 0 \le y \le 60\}.$$

$E_P \equiv \left(x^\#, y^\#\right) = (33.3803,\ 6.6658)$. The Fig. 6.1(a) illustrates the system stability behaviour of the interior equilibrium point of the model 2. Fig. 6.1(b) shows phase plane graph between herbivore species and leopard species in the model 2.

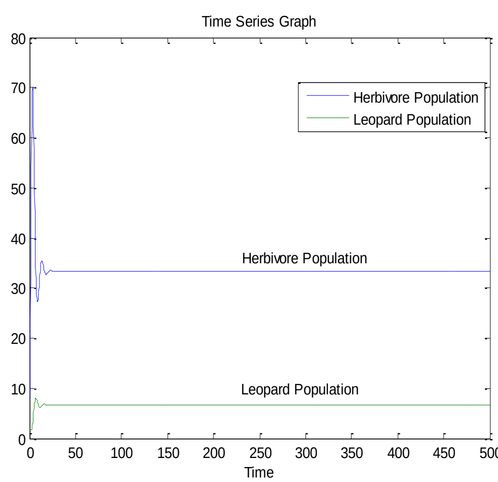

Fig. 6.1(a): Time series graph of the asymptotically stable interior equilibrium point $E_P(33.3803,\ 6.6658)$ of the model 2 with initial value $(8, 2)$.

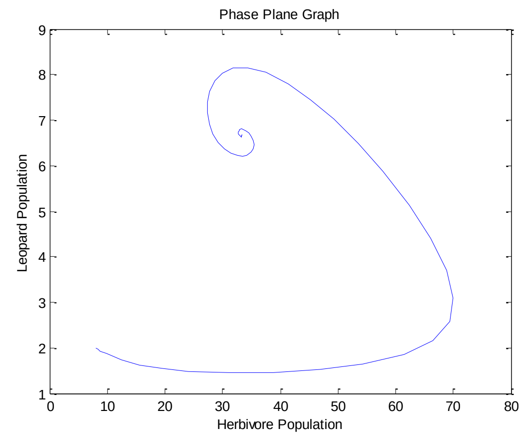

Fig. 6.1(b): Phase plane graph of the asymptotically stable interior equilibrium point $E_P(33.3803,\ 6.6658)$ of the model 2 with initial value $(8, 2)$.

Now we choose the following values of parameters in model 1

$r = 1.5; k = 100; a = 0.2; b = 0.15; e_1 = 0.1;$
$e_2 = 0.008; f = 0.05; d_1 = 0.2;\ d_2 = 0.2;$
$c = 0.01; m = 0.5; h = 0.1.$ (7.1)

Using above set of parameters given in (7.1), the stability region Ω_2 and interior equilibrium point E_{P_e} is given by

$$\Omega_2 = \{(x, y, z) \in R_+^3 : 0 \le x \le 100, 0 \le y \le 150, 0 \le z \le 12.5\}.$$

$E_{P_e} \equiv (x^*, y^*, z^*) = (23.7817,\ 3.8227,\ 2.9313)$. The Fig. 6.2(a) illustrates the system stability behaviour of the interior equilibrium point of the model 1. Fig. 6.2(b) shows phase plane graph between herbivore species, leopard species and exotic lion species in the model 1.

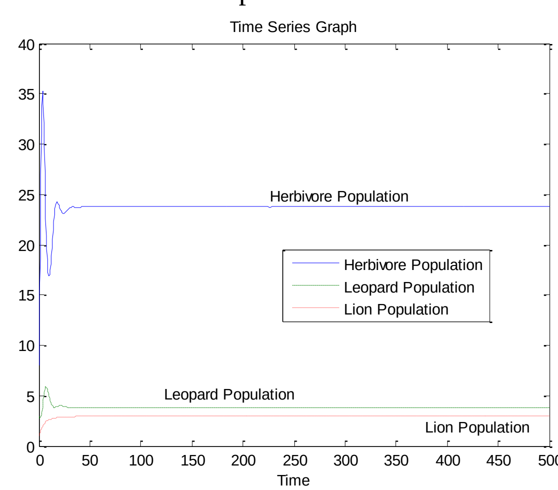

Fig. 6.2(a): Time series graph of the asymptotically stable interior equilibrium

point$E_{P_e}(23.7817,\ 3.8227,\ 2.9313)$ of the model 1 with initial value $(8, 3, 1)$.

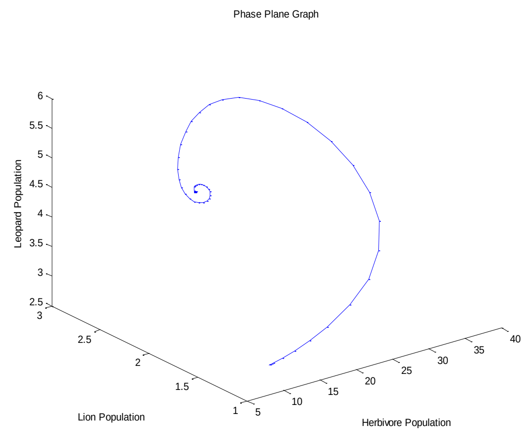

Fig. 6.2(b): Phase plane graph of the asymptotically stable interior equilibrium point $E_{P_e}(23.7817,\ 3.8227,\ 2.9313)$ of the model 1 with initial value $(8, 3, 1)$.

Now we explain the effect of changing of constant recruitment rate on account of Asiatic lion (exotic) on the model 1. Using distinct values of m as well as remaining parameters are same as given in table 6.1 of model 1, then behaviour of stable equilibrium points are given in following table 6.1.

Table 6.1: Behaviour of equilibrium point for distinct values of m.

m	**Behaviour of equilibrium point**
$0 < m < m_2^* = 0.888$	E_{B_e} is unstable E_{P_e} is stable
$m_2^* = 0.888 \leq m < m_1^* = 1.5$	E_{B_e} is stable E_{P_e} is unstable
$m \geq m_1^* = 1.5$	E_{B_e} is unstable E_{P_e} is unstable

(i) From Fig. 6.3, it may be noted that herbivore population and exotic lion species will survives and leopard population become extinct for $m = 1$

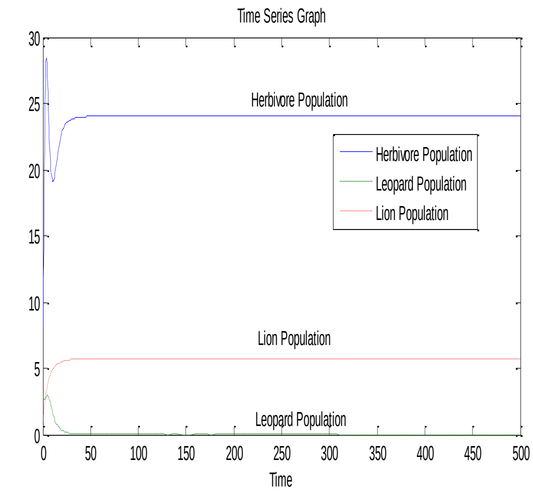

Fig. 6.3: Time series graph of the asymptotically stable boundary equilibrium point $E_{B_e}(24.0312,\ 0,\ 5.6987)$ of the model 1 with initial value $(8, 3, 1)$.

The Fig. 6.4(a), Fig. 6.4(b), and Fig. 6.4(c), illustrate the stable equilibrium level of herbivore population, Leopard population and Asiatic lion (Exotic species) population respectively for distinct values of m.

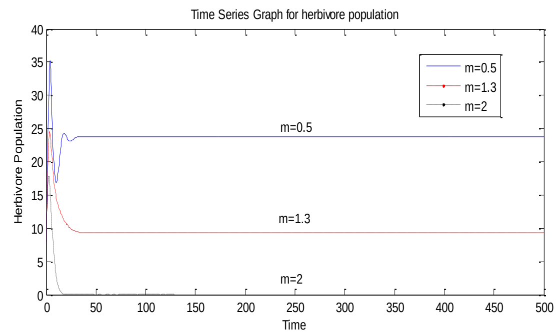

Fig. 6.4(a) Stable equilibrium level of herbivore population of the model 1 for distinct values of m.

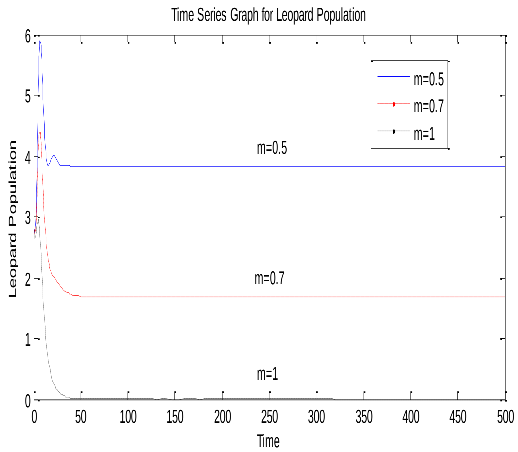

Fig. 6.4(b) Stable equilibrium level of leopard population of the model 1 for distinct values of m.

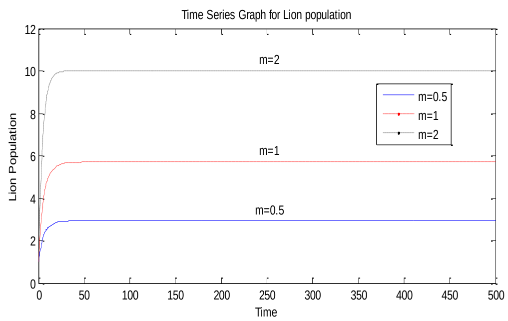

Fig. 6.4(c) Stable equilibrium level of lion population of the model 1 for distinct values of m.

CONCLUSION

In this paper, we have proposed a mathematical model to study the effect of Asiatic lion population on prey-predator system consisting native herbivore population and leopard population. From the stability of boundary equilibrium point E_{Be} [See Fig. 6.3], it is observed that the leopard population will not survive and consequently herbivore population will survive and exotic lion population will also survive. It may be observed from the stability conditions of interior equilibrium E_{P_e} that all the three species that is herbivore species, leopard species and exotic lion species would coexist if (i) ratio of predation rates of leopard to lion is greater than the ratio of conversion efficiencies of lion to leopard, (ii) product of constant recruitment rate of exotic lion population and predation rate of herbivore population by leopard population is greater than product of predation rate of herbivore population by exotic lion population, conversion efficiency and equilibrium level of exotic lion population, (iii) ratio of intra specific interference coefficient within exotic lion species to inter specific interference coefficient due to exotic lion species is greater than ratio of conversion efficiency of leopard population to death rate of leopard population, and (iv) the intra specific interference coefficient within exotic lion species is greater than the inter specific interference coefficient due to exotic lion species. From the global stability analysis of equilibrium point E_{P_e} we derived that E_{P_e} is more stable if interspecific interference coefficient due to exotic lion species as well as constant recruitment rate of lion population decreased. We observed that the positive interior equilibrium point E_{P_e} is stable for $0 < m < m_2^*$ (See Fig. 6.2(a), 6.2(b)). Boundary equilibrium point E_{P_e} is stable for $m_2^* \leq m < m_1^*$ (see Fig. 6.3). The switching in stability behaviour based on constant recruitment rate of lion population □ is also observed (See Fig. 6.2(a), 6.2(b), 6.3). Finally we conclude that if constant recruitment rate of exotic lion population is greater than some critical value m_1^* then it is harmful to native prey-predator system. The constant recruitment rate of lion population creates complex phenomena in the system.

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