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SOLUTION OF Nth ORDER FUZZY INITIAL VALUE PROBLEM

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ABSTRACT

In this paper we consider higher order linear differential equations with fuzzy initial values that occurs in almost all engineering branches. Here, We find solution for constant coefficient and variable coefficient third order linear differential equation by using method based on properties of linear transformation We show that fuzzy problem has unique solution if corresponding crisp Problem has unique solution. We will also prove that if the initial values are triangular fuzzy numbers, then the values of the solution at a given time are also triangular fuzzy numbers. We are going to propose a method to find fuzzy solution. We present three examples, one is homogeneous and another is non-homogeneous linear differential equation with constant coefficient and third is non-homogeneous linear differential equation with variable coefficient (Cauchy-Euler equation) to illustrate applicability of proposed method. We also plot graphs to show difference between exact and fuzzy solutions. This shows that our method is practical and applicable to solve nth order fuzzy initial value problems.

AMS Subject Classification code: 34A07: Fuzzy differential equations.
Keywords: Fuzzy initial value problem, Fuzzy set, Fuzzy number and linear transformation.

INTRODUCTION

Fuzzy initial value problem occurs in almost all Engineering branches. The term "Fuzzy differential equation" was put forward for the first time by Kandel and Byatt [1],The fuzzy initial value problem was studied by Seikkala [26].The fuzzy initial problem has been investigated by many authors so far Buckley and Feuring [7,8]; Buckley, Feuring, and Hayashi [9]; Lakshmikantham and Nieto [18]; Bede and Gal [5]; Bede, Rudas, and Bencsik [3]; Perfilieva et al.[25]; Chalco-Cano and Román-Flores[10,11]; Khastan, Bahrami, and Ivaz [14]; Gasilov, Amrahov, and Fatullayev [17]; Khastan,Nieto,Rodríguez-López [16]; Patel, Desai [22].Gasilo et al.[13],Gomes and Barros [4,28] proposed concepts of fuzzy calculus, analogically to classical calculus, and studied fuzzy differential equations in terms of this calculus. Under certain conditions, they established the existence of a solution for the first order fuzzy initial value problem and suggested a solution method. Gasilov et al. [17] benefitted from properties of linear transformations and proposed a method to find fuzzy bunch of Solution functions for linear equation. The method is applicable to higher order linear differential equations with constant coefficients.

Most of the researchers assume that derivative in the differential equation as a derivative of a fuzzy function in some sense. In earlier researches the derivative was considered as derivative. A study in this direction was made by Kaleva [19,20,21]. When Hukuhara derivative is used, then uncertainty of the solution may increase infinitely with time.

Furthermore, Bede and Gal [5] showed that a simple fuzzy function, generated by multiplication of differentiable crisp function and a fuzzy number, may not have Hukuhara derivative. In order to overcome this difficulty Bede and Gal [5] developed the generalized derivative concept and after that the studies about this subject were accelerated (Bede[2]); Bede et al [6]; Khastan and Nieto[15]; Chalco-Cano,Román-Flores[10,11,29]. But in case of generalized Hukuhara derivatives there are four different cases for second order fuzzy differential equation. Khastan A.,Gasilov.N.A., Fatullayev.A.G, Amrahov.S.E.[27], found solution of constant coefficient FBVP. Patel, Desai [23,24] solve fuzzy initial value problems by fuzzy Laplace transform.

In this paper we consider the fuzzy initial value problem as a set of crisp problem using properties of linear transformations. We solve higher order Cauchy-Euler's equation. For clarity we explain the proposed method for third order fuzzy linear differential equations, but the results are true for higher-order equations too. The fuzzy solution proposed by our method coincides with extension principle's results.

PRELIMINARIES

Definition: Membership function:

A fuzzy set $\tilde{A}$ can be defined as a pair of the universal set U and the membership function $\mu: U -> [0,1]$ for each $x \in U$, the number $\mu_{\tilde{A}}$ is called the membership degree of x in $\tilde{A}$.

Definition: $\alpha - cut\ set$:

$For\ each\ \alpha \in (0,1]$ the crisp set $A_\alpha = \{x \in U | \mu_{\tilde{A}}(x) \geq \alpha\}$ is called $\alpha - cut\ set$ of $\tilde{A}$.We use the notation $\tilde{u} = (u_{L,}(\alpha), u_R(\alpha))\ 0 \leq \alpha \leq 1$ to indicate a fuzzy number in parametric form. We denote $\underline{u} = u_L(0)$ and $\bar{u} = u_R(0)$ to indicate the left and the right end-points of $\tilde{u}$ respectively. an α-cut of $\tilde{u}$ is an interval $[u_{L,}(\alpha), u_R(\alpha)]$, which we denote as $u_\alpha = [\underline{u}\alpha, \bar{u}\alpha]$.

Definition:Fuzzy Number

A fuzzy number is a fuzzy set like $U: R \rightarrow I = [0,1]$ which satisfies:

(a) u is upper semi-continuous.

(b) u is fuzzy convex i.e.$(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}\ \forall x, y \in R, \lambda \in [0,1]$

(c) u is normal i.e $\exists x_0 \in R$ for which $u(x_0) = 1$

(d)$For\ each\ \alpha \in (0,1], supp\ u = \{x \in R | u_\alpha(x) > 0\}$ is support of u,and its $cl(supp\ u)$ is compact.

Definition: Triangular Fuzzy Number:

The Triangular fuzzy number as $\tilde{u}$= (a, b, c) for which $u_{L,}(\alpha)$ = a + α (b − a), $u_R(\alpha)$ = b - α (c − b) and $\underline{u}$ = a, $\bar{u}$= c. In geometric interpretations, we refer to the point b as a vertex.

Let us consider a triangular fuzzy number $\tilde{u}$ = (p, 0, q) the vertex of which is 0 (Note that p < 0 and q > 0 in this case).Then $u_L(\alpha)$ = (1−α) p and $u_R(\alpha)$ = (1−α) q and consequently, α-cuts are intervals

$$[(1-\alpha)\,p, (1-\alpha)\,q] = (1-\alpha)[p, q]$$

From the last representation one can see that an α-cut is similar to the interval [p, q] (i.e. to the 0-cut) with similarity coefficient (1−α).

We often express a fuzzy number $\tilde{u}$ as $\tilde{u} = u_{cr} + \tilde{u}_{un}$(crisp part + uncertainty). Here u_{cr} is a number with membership degree 1 and represents the crisp part (the vertex) of $\tilde{u}$,while $\tilde{u}_{un}$ represents the uncertain part with vertex at the origin.

For a triangular fuzzy number $\tilde{u}$= (a, b, c) we have $u_{cr} = b$ and $\tilde{u}_{un} = (a-b, 0, c-b)$.

Properties of Fuzzy Valued Number

For arbitrary $u = (\underline{u}(r), \bar{u}(r))\ v = (\underline{v}(r), \bar{v}(r))$, $0 \leq r \leq 1$ and arbitrary $k \in R$.

We define addition, subtraction, multiplication, scalar multiplication by k.

$$u + v = (\underline{u}(r) + \underline{v}(r), \bar{u}(r) + \bar{v}(r))$$
$$u - v = (\underline{u}(r) - \bar{v}(r), \bar{u}(r) - \underline{v}(r))$$

$$u \cdot v = (\min\{\underline{u}(r)\bar{v}(r), \underline{u}(r)\underline{v}(r), \bar{u}(r)\bar{v}(r), \bar{u}(r)\underline{v}(r)\}, \max\{\underline{u}(r)\bar{v}(r), \underline{u}(r)\underline{v}(r), \bar{u}(r)\bar{v}(r), \bar{u}(r)\underline{v}(r)\})$$

$$ku = \begin{cases} (k\underline{u}(r), k\bar{u}(r)) & if\ \ k \geq 0 \\ (k\bar{u}(r), k\underline{u}(r)) & if\ \ k < 0 \end{cases}$$

Definition:Hukuhara difference

Let $x, y \in E$.If there exists $z \in E$ such that $x = y + z$, then z is called the Hukuhara difference of fuzzy numbers x and y, and it is denoted by $z = x\ \Theta\ y$. The Θ sign stands for Hukuhara-difference and $x\,\Theta\,y \neq x + (-1)\,y$.

Definition:Hukuhara differential

Let $f: (a, b) \rightarrow E$ and $t_0 \in (a, b)$ if there exists an element $f'(t_0) \in$ E such that for all $\square > 0$ sufficiently small, exists $f(t_0 + \square)\ \Theta\ f(t_0)$, $f(t_0)\ \Theta f(t_0 - \square)$ and the limits holds(in the metric D)

$$\lim_{\square \to 0} \frac{f(t_0+h)\ \Theta\ f(t_0)}{\square} = \lim_{\square \to 0} \frac{f(t_0)\ \Theta\ f(t_0-h)}{\square} = f'(t_0)$$

Here derivative is considered as Hukuhara derivative.

FUZZY INITIAL VALUE PROBLEMS (FIVPS)

In this section, we have described a fuzzy initial value problem (FIVP) and concept of solution**.** We investigate a fuzzy Initial value problem with crisp linear differential equation and fuzzy initial values. FIVP can arise in modeling of a process the dynamics of which is crisp but there are uncertainties in initial values.

Consider the n^{th} order fuzzy initial value problem where $b_n(x) \neq 0$.

$$\left\{\begin{matrix} b_n(x)y^n + b_{n-1}(x)y^{n-1} + \cdots + b_1(x)y' + b_0(x)y = f(x) \\ y(l) = \widetilde{A_1} \\ y'(l) = \widetilde{A_2} \\ y''(l) = \widetilde{A_3} \\ \cdots \\ \cdots \\ y^{n-1}(l) = \widetilde{A_n} \end{matrix}\right\} \quad (1)$$

Where $\widetilde{A_1}, \widetilde{A_2}, \widetilde{A_3}, \ldots, \widetilde{A_n}$ are fuzzy numbers and $b_0(x), b_1(x), \ldots, b_n(x)$ and $f(x)$ are continuous crisp functions or constants and l is any integer.Let us represent the initial values as $\widetilde{A_1} = a_1 + \widetilde{a_1}$, $\widetilde{A_2} = a_2 + \widetilde{a_2}, \ldots, \widetilde{A_n} = a_n + \widetilde{a_n}$ where $a_1, a_2, \ldots, a_n$ are crisp numbers while $\widetilde{a_1}$ $\widetilde{a_2}, \ldots, \widetilde{a_n}$ are fuzzy numbers. We split the FIVP in Eq.(1) to the following problems:

Associated crisp problem (which is non-homogeneous)

$$\left\{\begin{matrix} b_n(x)y^n + b_{n-1}(x)y^{n-1} + \cdots + b_1(x)y' + b_0(x)y = f(x) \\ y(l) = a_1 \\ y'(l) = a_2 \\ y''(l) = a_3 \\ \cdots \\ \cdots \\ y^{n-1}(l) = a_n \end{matrix}\right\} \quad (2)$$

Homogeneous problem with fuzzy initial values

$$\left\{\begin{matrix} b_n(x)y^n + b_{n-1}(x)y^{n-1} + \cdots + b_1(x)y' + b_0(x)y = 0 \\ y(l) = \widetilde{a_1} \\ y'(l) = \widetilde{a_2} \\ y''(l) = \widetilde{a_3} \\ \cdots \\ \cdots \\ y^{n-1}(l) = \widetilde{a_n} \end{matrix}\right\} \quad (3)$$

It is easy to see if $y_{cr}(x)$ and $\tilde{y}_{un}(x)$ are solutions of Eq.(2) and Eq.(3) respectively then $\tilde{y}(x) = y_{cr}(x) + \tilde{y}_{un}(x)$ is a solution of the given problem in Eq.(1). Hence, Eq.(1) is reduced to solving a non-homogeneous equation with crisp conditions in Eq.(2) and homogeneous equation with fuzzy initial conditions in Eq.(3). Therefore, we will investigate how to solve Eq.(1).To determine $\tilde{y}(x)$ we consider Linear transformation $T: R^n \to R, T(u) = v\,u$, where v is fixed $n\,x\,n$ determinant and $u = [a_1, a_2, \ldots a_n]^T$.

THE SOLUTION ALGORITHM

The solution algorithm consists of four steps:

- Represent the initial values as $\widetilde{A_1} = a_1 + \widetilde{a_1}$, $\widetilde{A_2} = a_2 + \widetilde{a_2}, \ldots, \widetilde{A_n} = a_n + \widetilde{a_n}$
- Find linear independent solutions $y_1(x)$, $y_2(x), \ldots, y_n(x)$ of the crisp differential equation $b_n(x)y^n + b_{n-1}(x)y^{n-1} + \cdots b_1(x)y' + b_0(x)y = f(x)$.Constitute the vector-function $s(x) = (y_1(x), y_2(x), \ldots, y_n(x))$, the determinant W and calculate the vector-function $t(x) = s(x)W^{-1} = (t_1(x), t_2(x), \ldots, t_n(x))$ by formula at $x = l$.

  The Wronskain

  $$W = \begin{vmatrix} y_1(l) & y_2(l) & \cdots & y_n(l) \\ \vdots & \vdots & \ddots & \vdots \\ {y_1}^{n-1}(l) & {y_2}^{n-1}(l) & \cdots & {y_n}^{n-1}(l) \end{vmatrix}$$

  and $\det(W) \neq 0 \ \therefore W^{-1}\ exist.$
- Find the solution $y_{cr}(x)$ of the non-homogeneous crisp problem.
- The solution of the given problems
  For homogeneous FIVP
  $\tilde{y}(x) = t_1(x)\,\widetilde{a_1} + t_2(x)\,\widetilde{a_2} + \ldots + (x)\,\widetilde{a_n}$.
  For non-homogeneous FIVP
  $\tilde{y}(x) = y_{cr}(x) + t_1(x)\,\widetilde{a_1} + t_2(x)\,\widetilde{a_2} + \ldots + t_n(x)\,\widetilde{a_n}$.

EXAMPLES

Example:5.1 Solve the 3rd order FIVP with constant coefficient homogeneous equation.

$$\left\{\begin{matrix} y''' + 3y'' + 3y' + y = 0 \\ y(0) = (-0.5, 0, 1) \\ y'(0) = (-1, 0, 1) \\ y''(0) = (-1, 0, 0.5) \end{matrix}\right\} \quad (4)$$

The problem is homogeneous and initial values are fuzzy numbers with vertices at 0.therefore solution by solution algorithm.$y(x) = e^{-x}$, $y_2(x) = xe^{-x}$and $y_3(x) = x^2e^{-x}$are linearly independent solution for the equation $y''' + 3y'' + 3y' + y = 0$.

Hence $s(x) = (e^{-x}, xe^{-x}, x^2e^{-x})$ and

$$W = \begin{vmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & -2 & 2 \end{vmatrix} \quad \text{and}$$

$$t(x) = s(x)W^{-1} = (t_1(x), t_2(x), t_3(x)),$$

$$t(x) = (e^{-x} + xe^{-x} + \frac{x^2}{2}e^{-x}, xe^{-x} + x^2e^{-x}, \frac{x^2}{2}e^{-x})$$

$$\tilde{y}_{un}(x) = (e^{-x} + xe^{-x} + x^2e^{-x})(-0.5,0,1) + (xe^{-x} + x^2e^{-x})(-1,0,1) + \frac{x^2}{2}e^{-x}(-1,0,0.5)$$

Where the arithmetic operations are considered to be fuzzy operations.

The fuzzy solution $\tilde{y}(x)$ form band in the xy-coordinate space (Fig.1)

Since the initial values are triangular fuzzy numbers, an α-cut of the solution can be determined by similarity coefficient (1−α), i.e.

$y_\alpha(x) = (1-\alpha)[\underline{y}(x), \overline{y}(x)]$

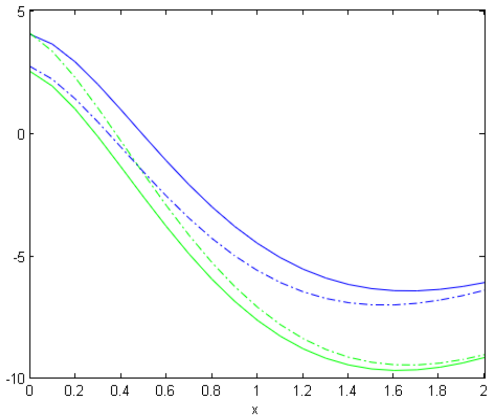

Fig.1 The fuzzy solution, obtained by the proposed method, for Example 5.1

Example:5.2 Consider the 3rd order FIVP with constant coefficient non-homogeneous equation.

$$\begin{Bmatrix} y''' + 3y'' + 3y' + y = 30e^{-x} \\ y(0) = (2.5,3,4) \\ y'(0) = (-4,-3,-2) \\ y''(0) = (-48,-47,-46.5) \end{Bmatrix} \qquad (5)$$

We represent the initial values as

$\tilde{A}$= (2.5,3,4) = 3 + (-0.5,0,1),
$\tilde{B}$= (-4,-3,-2) = -3 + (-1,0,1),
$\tilde{C}$=(-48,-47,-46.5) = - 47 + (-1,0,0.5).

we solve crisp non-homogeneous crisp problem

$$\begin{Bmatrix} y''' + 3y'' + 3y' + y = 30e^{-x} \\ y(0) = 3 \\ y'(0) = -3 \\ y''(0) = -47 \end{Bmatrix} \qquad (6)$$

And the crisp solution

$$y_{cr}(x) = (3 - 25x^2)e^{-x} + 5x^3e^{-x}$$

$$y_{cr}(x) = 3(e^{-x} + xe^{-x} + x^2e^{-x}) - 3(xe^{-x} + x^2e^{-x}) - \frac{47x^2}{2}e^{-x} + 5x^3e^{-x}$$

Fuzzy homogeneous problem to find the uncertainty of the solution is as follows:

$$\begin{Bmatrix} y''' + 3y'' + 3y' + y = 0 \\ y(0) = (0.5,0,1) \\ y'(0) = (-1,0,1) \\ y''(0) = (-1,0,0.5) \end{Bmatrix} \qquad (7)$$

This problem is the same as Example 1. Hence, the solution is

$$\tilde{y}_{un}(x) = (e^{-x} + xe^{-x} + x^2e^{-x})(-0.5,0,1) + (xe^{-x} + x^2e^{-x})(-1,0,1) + \frac{x^2}{2}e^{-x}(-1,0,0.5)$$

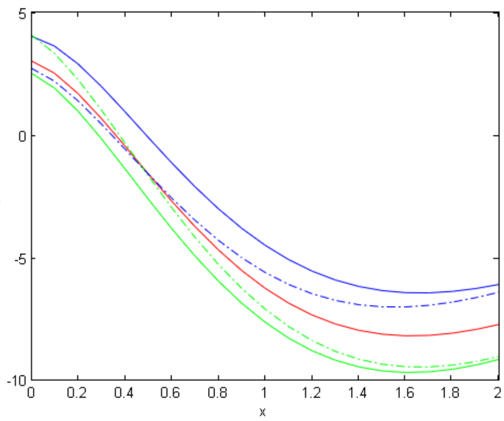

Fig. 2 The fuzzy solution $\tilde{y}(x)$, obtained by the proposed method, for Example 5.2.Red line represents the crisp solution.

We add this uncertainty to the crisp solution and get the fuzzy solution of the given FIVP

$$\tilde{y}(x) = y_{cr}(x) + \tilde{y}_{un}(x)$$

$$= (e^{-x} + xe^{-x} + x^2e^{-x})(2.5,3,4)$$
$$+ \ (xe^{-x} + x^2e^{-x})(-4,-3,-2)$$
$$+\frac{x^2}{2}e^{-x}(\text{-48,-47,-46.5}) + 5x^3e^{-x}$$

In above example if we take $x \to \infty$ then fuzziness in solution is disappeared. So the solution goes nearer to zero if we increase x.(see Fig.3)

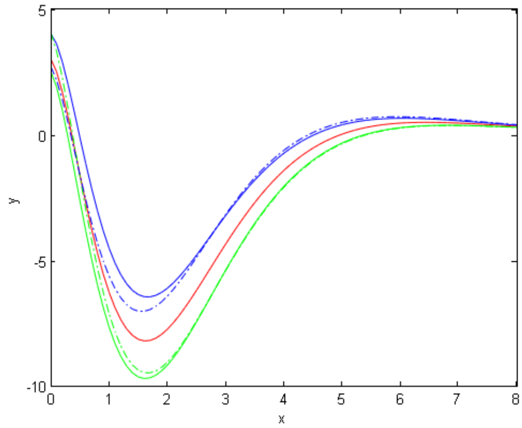

Fig. 3 If x increases fuzziness disappears

Example:5.3 Consider 3rd order FIVP with variable coefficient.

$$\begin{cases} (x^3D^3 - 3x^2D^2 + 6xD - 6)y = \frac{12}{x} \\ y(1) = (4,5,6) \\ y'(1) = (12,13,14.5) \\ y''(1) = (9.5,10,11) \end{cases} \quad (8)$$

$$\begin{cases} (x^3D^3 - 3x^2D^2 + 6xD - 6)y = \frac{12}{x} \\ y(1) = 5 \\ y'(1) = 13 \\ y''(1) = 10 \end{cases} \quad (9)$$

Associated non-homogeneous problem has the solution i.e. exact solution of non-homogeneous problem

$$y_{cr}(x) = \frac{-138}{24}x + \frac{328}{24}x^2 + \frac{-69}{24}x^3 - \frac{1}{24x}$$

Consider homogeneous equation

$$\begin{cases} (x^3D^3 - 3x^2D^2 + 6xD - 6)y = 0 \\ y(1) = (-1,0,1) \\ y'(1) = (-1,0,0.5) \\ y''(1) = (-0.5,0,1) \end{cases} \quad (10)$$

The problem is homogeneous and initial values are fuzzy numbers with vertices at 0.therefore solution by solution algorithm.$y_1(x) = x$, $y_2(x) = x^2$ and $y_2(x) = x^3$ are linearly independent solution of equation

$$(\, x^3D^3 - 3x^2D^2 + 6xD - 6)y = 0$$

Hence $s(x)$ =$(x, x^2 \ \ , x^3 \ )$ and

$$W = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix}$$

And

$t(x)$= $s(x)W^{-1} = (t_1(x), t_2(x), t_3(x))$

$$t(x) = \left(3x - 3x^2 + x^3, -2x + 3x^2 - x^3, \frac{x}{2} - x^2 + \frac{x^3}{2}\right)$$

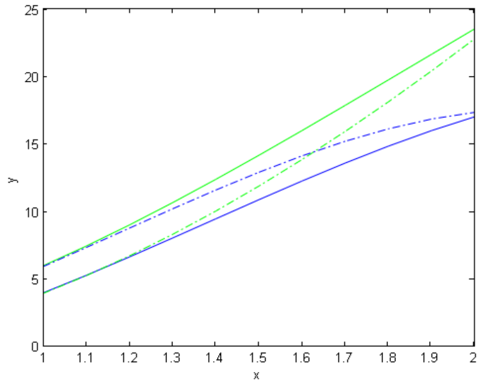

Fig.4 The fuzzy solution, obtained by the proposed method, for Example 5.3

The fuzzy solution is

$$\tilde{y}_{un}(x) = (3x - 3x^2 + x^3\,)(-1,0,1)$$
$$+(-2x + 3x^2 - \ x^3)(-1,0,0.5)$$
$$+ \ (\frac{x}{2} - x^2 + \frac{x^3}{2})(-0.5,0,1)$$

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We add this uncertainty to the crisp solution and get the fuzzy solution of the given FIVP

$$\tilde{y}(x) = y_{cr}(x) + \tilde{y}_{un}(x)$$
$$= (3x - 3x^2 + x^3)(4,5,6)$$
$$+(-2x + 3x^2 - x^3) \quad (12, 13, 14.5)$$
$$+(\frac{x}{2} - x^2 + \frac{x^3}{2}(9.5,10,11) - \frac{1}{24x}$$

$$\tilde{y}(x) = -6x + 14x^2 - 3x^3 - \frac{1}{24x}$$

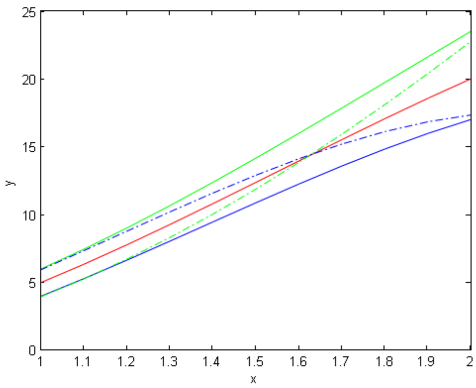


Fig.5 The fuzzy solution, obtained by the proposed method, for Example 5.3.Red line represents the crisp solution.

In above example if we increase x and as we take $x \to \infty$ then fuzziness in solution increases and goes to infinite.

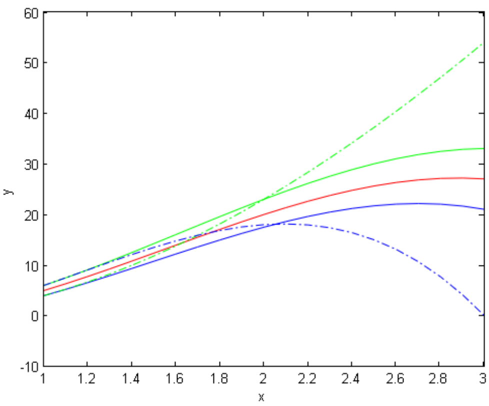
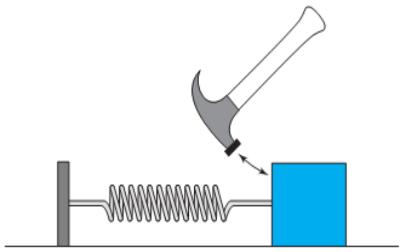


Fig. 6 If we increase x fuzziness increases

Example:5.4 Consider a unit mass sliding on a frictionless table attached to a spring, with spring constant k =16. Suppose the mass is lightly tapped by a hammer every T seconds. Suppose that the first tap occurs at time t = 0 and before that time the mass is at rest. Describe what happens to the motion of the mass for the tapping period T =1.



Mass tapped periodically with a hammer.

Fig. 7

$$\left\{\begin{array}{c} x''(t) + 16\,x(t) = sint \\ x(0) = 0 \\ x'(0) = 1 \end{array}\right\} \quad (11)$$

Consider the homogeneous equation with fuzzy parameters.

$$\left\{\begin{array}{c} x''(t) + 16\,x(t) = 0 \\ x(0) = (-1,0,1) \\ x'(0) = (-0.5,0,1) \end{array}\right\} \quad (12)$$

The problem is homogeneous and initial values are fuzzy numbers with vertex 0 therefore solution by solution algorithm. $x_1(t) = cos4t\,, x_2(t) = sin4t$ are linearly independent solution for the equation $x''(t) + 16x(t) = 0$

Hence $s(t) = (cos4t, sin4t)$ and

$$W = \begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix}$$

and $y(t) = (y_1(t), y_2(t))$
Here $y(t) = (cos4t, (1/4)\, sin4t)$
$\tilde{x}_{un}(t) = (\cos 4t)(-1,0,1) + (1/4)\sin 4t(-0.5,0,1)$
Crisp solution of given non-homogeneous problem
$x_{cr}(t) = (7/30)sin4t + (1/15)sint$.

$$\tilde{x}(t) = x_{cr}(t) + \tilde{x}_{un}(t)$$

$=cos4t(-1,0,1)+(7/30)sin4t(0.5,1,2)$
$+ (1/15)sint = (7/30)sin4t + (1/15)sint$

where arithmetic operations are considered to be fuzzy operations. the fuzzy solution $\tilde{x}(t)$form band in the tx-coordinate space.(Fig.8)

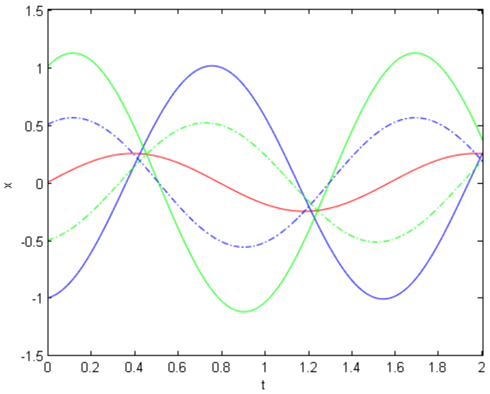


Fig.8:The periodic fuzzy solution by properties of Linear transformation

If we apply periodic force to block, it will move in given open interval that we denote by triangular fuzzy number, Here the damping force is negligible. The spring mass system occurs in all most all engineering branches as well as in the many real life situations.

CONCLUSIONS

In this paper we have represented the fuzzy initial value problems as a set of crisp problems. We have proposed a solution method based on the properties of linear transformations. For clarity we have explained the proposed method for third order linear differential equation but it is applicable for nth order also. Here we have solved third order Cauchy-Euler equation i.e. we solved variable coefficient initial value problems based on the properties of linear transformations. We also solved one application level problem.

AIM AND SCOPE OF THE WORK

The aim of paper is to find solution of any Fuzzy differential equation having uncertain initial conditions. In future we are going to solve Fuzzy differential equations having the applications in different branches of engineering. we will solve higher order FBVP based on the properties of linear transformations.

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