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MATHEMATICAL MODELLING OF FINGERO-IMBIBITION PHENOMENON IN HETEROGENEOUS POROUS MEDIUM WITH MAGNETIC FIELD EFFECT

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ABSTRACT

The present paper discusses the mathematical model for fingero-imbibition phenomenon arising in fluid flow through the heterogeneous porous medium with magnetic field effect during secondary oil recovery process. The mathematical formulation leads to a nonlinear partial differential equation and its solution has been obtained with appropriate boundary conditions by homotopy analysis method. The solution represents the saturation of injected water for fingero-imbibition phenomenon which increases when distance increases for given time. The graphical and numerical representations of the solution are discussed.

Keywords: Fluid flow, Heterogeneous porous medium, Fingero-imbibition phenomenon, Homotopy analysis method.

AMS subject classification: 76S05, 76Txx, 65Nxx, 35Q35.

INTRODUCTION

If a porous medium filled with some phase (oil) is brought into contact with another phase (water) which preferentially wets the medium, there is a spontaneous flow of the wetting phase (water) into the medium and a counter flow of the native phase (oil) from the medium. This phenomenon occurring due to the difference of wetting abilities of the phases is called imbibition phenomenon. Besides this if a porous medium filled with one phase (oil) is displaced by another phase (water) of lesser viscosity, then instead of regular displacement of the whole front, protuberances may occur which shoot through the porous medium at relatively very high speed giving rise to the fingering phenomenon. This simultaneous occurrence of both phenomena fingering and imbibition is known as fingero-imbibition phenomenon (see fig. 1).

The imbibition phenomenon have been investigated by many researches with different aspects. Mishra and Verma [3] have discussed imbibition in the flow of two immiscible fluids (oil and water) with magnetic field. Shah and Verma [5] have obtained the numerical solution of fingero-imbibition phenomenon through homogeneous porous media with magnetic field using finite difference method. Desai [21] has discussed the imbibition phenomenon by similarity transform. Patel, Mehta and Patel [9] have discussed mathematical model of imbibition phenomenon in heterogeneous porous media. Parikh, Mehta and Pradhan [10] have discussed mathematical modeling of fingero-imbibition phenomenon in homogeneous porous medium with magnetic field effect in vertical downward direction. Patel, Rabari and Bhathawala [11] have obtained numerical solution of imbibition phenomenon in a homogeneous medium with magnetic fluid.

In the present work, we have developed the mathematical model for fingero-imbibition

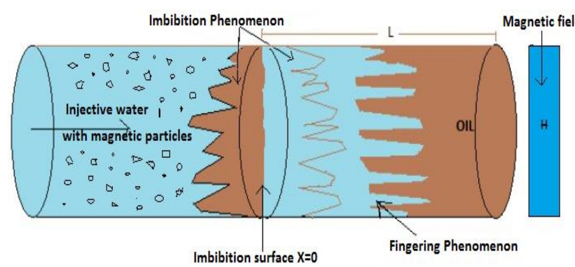

Figure 1: Fingero-imbibition phenomenon.

phenomenon in heterogeneous porous medium with magnetic field effect. The mathematical formulation leads to the governing nonlinear partial differential equation. The solution of the problem have been obtained using homotopy analysis method [20]. The main aim of this work is to find the solution (the saturation of injected water) of the fingero-imbibition phenomenon in the heterogeneous porous medium with magnetic field effect.

AIM AND SCOPE OF THE WORK

The aim with this work is to discuss the mathematical model for fingero-imbibition phenomenon arising in fluid flow through the heterogeneous porous medium with magnetic field effect during secondary oil recovery process. The solution of problem is obtain by using homotopy analysis method. This solution represents the saturation of injected water which helps us to predict the amount of water required to inject for recovering oil. This type of mathematical model is useful for predicting oil recovery from petroleum reservoir.

STATEMENT OF THE PROBLEM

Here we consider the cylindrical piece of heterogeneous porous matrix of length L which is filled with oil. During the secondary oil recovery process, the imbibition phenomenon will occur simultaneously with fingering which describes the fingero-imbibition phenomenon. It is considered that the flow of water with magnetic particles and oil in the heterogeneous porous medium under the variable magnetic field effect. In this work assumed that the injected water is conductive while the oil is non-conductive and the effect of a variable magnetic field is to increase the velocity of injected water by gradient of $\frac{\omega H^2}{8\pi}$ where ω is permeability of magnetic field H. It is assumed that the Darcy's law is valid for the investigated flow system for the mathematical model of the fingero-imbibition phenomenon and the average cross sectional area occupied by the fingers was observed.

The saturation of the injected water $S_w(x,t)$ is then defined as the average cross-sectional area occupied by injected water at distance x and time t. The porosity and permeability of heterogeneous porous medium may vary from one place to another place. Considered that the porosity and permeability of heterogeneous porous medium are the functions of variable x only.

MATHEMATICAL MODEL

The velocity of injected water V_w and velocity of oil V_o can be represented due to Darcy's law as [14, 15]:

$$V_w = -\frac{k_w}{\delta_w} K \left[\frac{\partial P_w}{\partial x} + \frac{\omega H}{4\pi}\frac{\partial H}{\partial x}\right] \tag{1}$$

$$V_o = -\frac{k_o}{\delta_o} K \frac{\partial P_o}{\partial x} \tag{2}$$

where V_w and V_o are the velocities of water and oil respectively, k_w and k_o are the relative permeabilities of water and oil respectively, δ_w and δ_o are the constant viscosities of water and oil respectively, K is the variable permeability of the heterogeneous porous medium, P_w and P_o are the pressures of water and oil respectively, ω is permeability of magnetic field H.

The equation of continuity of injected water is

$$P\frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \tag{3}$$

where $P = P(x)$ is the variable porosity of the heterogeneous porous medium.

The pressure difference is given by the capillary pressure P_c:

$$P_c = P_o - P_w\,. \tag{4}$$

The imbibition condition for countercurrent imbibition phenomenon can be expressed as [14]

$$V_w = -V_o\,. \tag{5}$$

Using equations (1), (2) and (4) in (5), we get

$$\frac{\partial P_w}{\partial x} = -\left(\frac{k_w}{\delta_w} + \frac{k_o}{\delta_o}\right)^{-1}\left(\frac{k_o}{\delta_o}\frac{\partial P_c}{\partial x} + \frac{k_w}{\delta_w}\frac{\omega H}{4\pi}\frac{\partial H}{\partial x}\right). \tag{6}$$

According to Scheidegger [14], we have

$$\frac{k_w}{\delta_w}\frac{k_o}{\delta_o}\left(\frac{k_w}{\delta_w} + \frac{k_o}{\delta_o}\right)^{-1} \approx \frac{k_o}{\delta_o}. \tag{7}$$

On substituting the value of $\frac{\partial P_w}{\partial x}$ with (7) in equation (1), we get

$$V_w = K\frac{k_o}{\delta_o}\left[\frac{\partial P_c}{\partial x} - \frac{\omega H}{4\pi}\frac{\partial H}{\partial x}\right]. \tag{8}$$

Substituting equation (8) into (3), we get

$$P\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[K\frac{k_o}{\delta_o}\frac{\partial P_c}{\partial x} - K\frac{k_o}{\delta_o}\frac{\omega H}{4\pi}\frac{\partial H}{\partial x}\right] = 0. \tag{9}$$

We assume that the capillary pressure P_c is a continuous linear function of the form [19]

$$P_c = -\beta S_w \tag{10}$$

where β is a constant.

Due to Scheidegger and Johnson [1], we assume the standard relationship between phase saturation and relative permeability as

$$k_w = S_w \text{ and } k_o = 1 - \alpha S_w \tag{11}$$

where α is a constant.

For the heterogeneous porous medium, we assume the porosity and permeability as functions of x only [2],

$$P(x) = \frac{1}{a_1 - a_2 x} \text{ and } K(x) = K_0(1 + bx) \tag{12}$$

where a_1, a_2, K_0 and b are positive constants. Since $P(x)$ can't exceed unity, we assume that $a_1 - a_2 x \geq 1$.

For simplicity, we consider $K \propto P$ [17],

$$K = K_c P \tag{13}$$

where K_c is a constant.

Considering the magnetic field in the x- direction only, we write H as [4, 22]

$$H = \lambda x^n \tag{14}$$

where λ is a constant parameter and n is an integer.

Using the value of H for $n = 1$ in equation (9) with equations (13), (11) and (10), we get

$$\frac{\partial S_w}{\partial t} = \frac{\beta K_c}{\delta_o P}\frac{\partial}{\partial x}\left[P(1-\alpha S_w)\frac{\partial S_w}{\partial x}\right] + \frac{K_c \omega \lambda^2}{4\pi \delta_o P}\frac{\partial}{\partial x}[P(1-\alpha S_w)x]. \tag{15}$$

Using dimensionless variables

$$X = \frac{x}{L}, \qquad T = \frac{\beta K_c t}{\delta_o L^2},$$

equation (15) becomes,

$$\frac{\partial S_w}{\partial T} = \frac{1}{P}\frac{\partial}{\partial X}\left[P(1-\alpha S_w)\frac{\partial S_w}{\partial X}\right] + \frac{\omega \lambda^2 L^2}{4\pi \beta P}\frac{\partial}{\partial X}[P(1-\alpha S_w)X]. \tag{16}$$

Now,

$$\frac{1}{P}\frac{\partial P}{\partial X} = \frac{\partial(\log P)}{\partial X}$$

$$= \frac{\partial}{\partial X}\left(\frac{a_2 L X}{a_1} - \log a_1\right)$$

$$(neglecting\ higher\ order\ terms\ of\ X)$$

$$= \frac{a_2 L}{a_1}.$$

Equation (16) reduces to

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X}\left[(1-\alpha S_w)\frac{\partial S_w}{\partial X}\right] + A(1-\alpha S_w)\frac{\partial S_w}{\partial X} + B\frac{\partial}{\partial X}[(1-\alpha S_w)X] + AB(1-\alpha S_w)X \tag{17}$$

where $A = \frac{a_2 L}{a_1}$, $B = \frac{\omega \lambda^2 L^2}{4\pi\beta}$ and $S_w(x,t) = S_w(X,T)$.

A set of suitable boundary conditions for fingero-imbibition phenomenon are considered as

$$S_w(0,T) = \frac{T}{5} \text{ and } S_w(1,T) = \frac{1+3T}{5} \tag{18}$$

The equation (17) is the desired governing nonlinear partial differential equation for the fingero-imbibition phenomenon in the heterogeneous porous medium with magnetic field effect. The solution $S_w(X,T)$ of equation (17) represents the saturation of injected water at distance X and time T.

APPLICATION OF HOMOTOPY ANALYSIS METHOD

In 1992, Liao [20] proposed a new technique homotopy analysis method (HAM) to obtain solutions of nonlinear differential equations. Many authors have applied the HAM for solving ordinary differential equations and partial differential equations. For example, Liao [16] has discussed solution of various ODEs by HAM. Liao [6] has obtained solution for an unsteady boundary-layer flow due to an impulsively stretched sheet by HAM. Ali and Mehmood [7] have discussed the solution of the unsteady boundary layer flow equations by HAM. Darvishi and Khani [8] have applied the HAM to solve the foam drainage equation. Patel and Desai [12, 13, 23] have applied the HAM to one dimensional partial differential equation arising in fluid flow through porous medium.

Let us consider the nonlinear partial differential equation according to equation (17) as

$$\mathcal{N}[\varphi(X,T;q)] = 0 \qquad (19)$$

where $q \in [0,1]$ is the embedding parameter, $\varphi(X,T;q)$ is an unknown function and a nonlinear operator $\mathcal{N}$ is defined as

$$\begin{aligned}\mathcal{N}[\varphi(X,T;q)] = {} & \frac{\partial^2\varphi(X,T;q)}{\partial X^2} \\ & - \alpha\varphi(X,T;q)\frac{\partial^2\varphi(X,T;q)}{\partial X^2} \\ & - \alpha\left\{\frac{\partial\varphi(X,T;q)}{\partial X}\right\}^2 \\ & + (A - B\alpha X)\frac{\partial\varphi(X,T;q)}{\partial X} \\ & - A\alpha\varphi(X,T;q)\frac{\partial\varphi(X,T;q)}{\partial X} \\ & - (B\alpha + AB\alpha X)\varphi(X,T;q) \\ & + ABX + B - \frac{\partial\varphi(X,T;q)}{\partial T} \qquad (20)\end{aligned}$$

According to boundary conditions (18), it is straightforward to choose initial approximation as

$$S_{w_0}(X,T) = \frac{T + X^2 + T(X + X^2)}{5} \qquad (21)$$

Now we choose the linear operator as

$$\mathcal{L}[\varphi(X,T;q)] = \frac{\partial^2\varphi(X,T;q)}{\partial X^2} \qquad (22)$$

which has the property $\mathcal{L}(f) = 0$ when $f = 0$.

Let $c_0 \neq 0$ be the convergence control parameter and $H(X,T)$ be a non-zero auxiliary function. Liao [20] constructed, the so-called zeroth-order deformation equation

$$\begin{aligned}(1-q)\mathcal{L}\big[\varphi(X,T;q) - S_{w_0}(X,T)\big] \\ = qc_0H(X,T)\mathcal{N}[\varphi(X,T;q)] \qquad (23)\end{aligned}$$

where $S_{w_0}(X,T)$ is an initial approximation of $S_w(X,T)$. When $q = 0$ and $q = 1$, we have

$$\varphi(X,T;0) = S_{w_0}(X,T) \text{ and } \varphi(X,T;1) = S_w(X,T) \qquad (24)$$

respectively. Therefore, when q increases from 0 to 1, the solution $\varphi(X,T;q)$ deforms (varies) from the initial approximation $S_{w_0}(X,T)$ to the solution $S_w(X,T)$. By Taylor's theorem, we expand $\varphi(X,T;q)$ in powers of q as

$$\varphi(X,T;q) = S_{w_0}(X,T) + \sum_{m=1}^{\infty} S_{w_m}(X,T)q^m \qquad (25)$$

where

$$S_{w_m}(X,T) = \frac{1}{m!}\frac{\partial^m\varphi(X,T;q)}{\partial q^m}\bigg|_{q=0}. \qquad (26)$$

Assume that the linear operator, the initial approximation, the convergence control parameter and the auxiliary function are selected such that the series (25) is convergent at $q = 1$. Then at $q = 1$, the series (25) becomes

$$S_w(X,T) = S_{w_0}(X,T) + \sum_{m=1}^{\infty} S_{w_m}(X,T) \qquad (27)$$

Define the vector $\overrightarrow{S_{w_n}} = \{S_{w_0}(X,T), S_{w_1}(X,T), \ldots, S_{w_n}(X,T)\}$. Differentiating (23) m times with respect to q and then putting $q = 0$ and finally dividing them by $m!$, we have the so-called high-order deformation equation

$$\begin{aligned}\mathcal{L}\big[S_{w_m}(X,T) - \chi_m S_{w_{m-1}}(X,T)\big] \\ = c_0H(X,T)\mathcal{R}_m\big(\overrightarrow{S_{w_{m-1}}}\big) \qquad (28)\end{aligned}$$

where

$$\begin{aligned}\mathcal{R}_m\big(\overrightarrow{S_{w_{m-1}}}\big) = {} & \frac{\partial^2 S_{w_{m-1}}}{\partial X^2} - \alpha\sum_{j=0}^{m-1} S_{w_j}\frac{\partial^2 S_{w_{m-1-j}}}{\partial X^2} \\ & - \alpha\sum_{j=0}^{m-1}\frac{\partial S_{w_j}}{\partial X}\frac{\partial S_{w_{m-1-j}}}{\partial X} \\ & + A\frac{\partial S_{w_{m-1}}}{\partial X} - B\alpha X\frac{\partial S_{w_{m-1}}}{\partial X} \\ & - A\alpha\sum_{j=0}^{m-1} S_{w_j}\frac{\partial S_{w_{m-1-j}}}{\partial X} \\ & - B\alpha S_{w_{m-1}} - AB\alpha X S_{w_{m-1}} \\ & + (B + ABX)(1-\chi_m) \\ & - \frac{\partial S_{w_{m-1}}}{\partial T}, m \geq 1 \qquad (29)\end{aligned}$$

and

$$\chi_m = \begin{cases} 0, & when \quad m \leq 1 \\ 1, & when \quad m > 1. \end{cases} \qquad (30)$$

Here we consider the auxiliary function as $H(X,T) = 1$. Then the equation (28) becomes

$$\begin{aligned} S_{w_m}(X,T) = {} & \chi_m S_{w_{m-1}}(X,T) \\ & + c_0 \mathcal{L}^{-1}\left[\mathcal{R}_m\left(\overrightarrow{S_{w_{m-1}}}\right)\right] + C_1 X \\ & + C_2 \end{aligned} \tag{31}$$

where C_1 and C_2 are determined by the boundary conditions $S_{w_m}(0,T)=0$ and $S_{w_m}(1,T)=0$, $m \geq 1$. Solution of (31) gives $S_{w_1}(X,T), S_{w_2}(X,T)$ and so on. Hence the homotopy series solution of (17) is as

$$\begin{aligned} S_w(X,T) = {} & \frac{T+X^2+T(X+X^2)}{5} + c_0\Bigg[-\frac{X}{20}+\frac{X^2}{10} \\ & -\frac{X^3}{30}-\frac{X^4}{60}-\frac{TX}{5}+\frac{TX^2}{5}-\frac{BX}{2} \\ & +\frac{BX^2}{2}-\frac{ABX}{6}+\frac{ABX^3}{6} \\ & +\frac{A}{5}\left(-\frac{X}{3}+\frac{X^3}{3}-\frac{5TX}{6}+\frac{TX^2}{2}\right. \\ & \left.+\frac{TX^3}{3}\right)-\frac{\alpha}{25}\left(-\frac{X}{2}+\frac{X^4}{2}-3TX\right. \\ & +TX^2+TX^3+TX^4-3T^2X \\ & \left.+\frac{3T^2X^2}{2}+T^2X^3+\frac{T^2X^4}{2}\right) \\ & -\frac{A\alpha}{25}\left(-\frac{X}{10}+\frac{X^5}{10}-\frac{47TX}{60}+\frac{TX^3}{3}\right. \\ & +\frac{TX^4}{4}+\frac{TX^5}{5}-\frac{27T^2X}{20}+\frac{T^2X^2}{2} \\ & \left.+\frac{T^2X^3}{2}+\frac{T^2X^4}{4}+\frac{T^2X^5}{10}\right) \\ & -\frac{B\alpha}{5}\left(-\frac{X}{4}+\frac{X^4}{4}-\frac{13TX}{12}+\frac{TX^2}{2}\right. \\ & \left.+\frac{TX^3}{3}+\frac{TX^4}{4}\right)-\frac{AB\alpha}{5}\left(-\frac{X}{20}\right. \\ & +\frac{X^5}{20}-\frac{3TX}{10}+\frac{TX^3}{6}+\frac{TX^4}{12} \\ & \left.\left.+\frac{TX^5}{20}\right)\right]+\cdots \end{aligned} \tag{32}$$

The solution (32) represents the saturation of injected water at distance X and time T for the fingero-imbibition phenomenon arising in fluid flow through the heterogeneous porous medium with magnetic field effect.

As pointed out by Liao, the convergence of the homotopy series solution depends upon the value of convergence control parameter c_0. The proper value of c_0 has been obtained using c_0-curves. Many authors have discussed the convergence of the homotopy series solution. For example, Liao [6, 16], Ali and Mehmood [7], Darvishi and Khani [8], Patel and Desai [12, 13, 23] have chosen a proper value of c_0 providing us the convergent homotopy series solution of nonlinear ODEs and PDEs. With the help of Mathematica package for nonlinear BVPs [18], the so-called c_0-curves are plotted for 20th order approximation. This helps us to discover the range for the admissible values of c_0, which corresponds to the horizontal line segment. It is obvious that the valid domain of c_0 is $-1.2 \leq c_0 \leq -0.4$ from the c_0-curves (see figures 2-4). This means that the series (32) converges for these values of c_0.

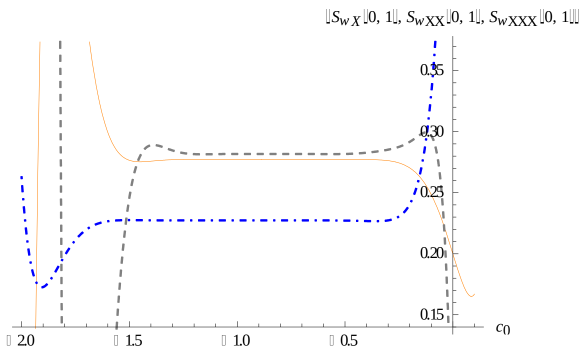

Figure 2: The c_0-curves of $S_{w_X}(0,1)$ (Solid line), $S_{w_{XX}}(0,1)$ (DotDashed line) and $S_{w_{XXX}}(0,1)$ (Dashed line).

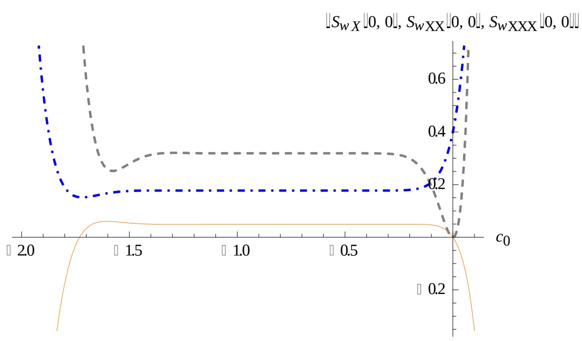

Figure 3: The c_0-curves of $S_{w_X}(0,0)$ (Solid line), $S_{w_{XX}}(0,0)$ (DotDashed line) and $S_{w_{XXX}}(0,0)$ (Dashed line).

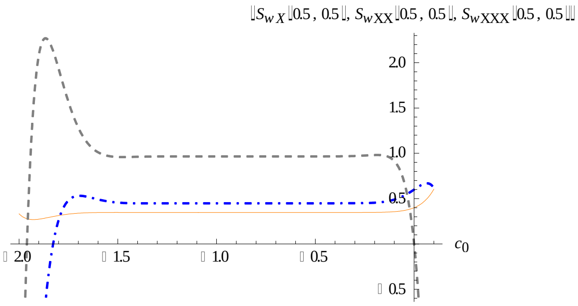

Figure 4: The c_0-curves of $S_{w_X}(0.5, 0.5)$ (Solid line), $S_{w_{XX}}(0.5, 0.5)$ (DotDashed line) and $S_{w_{XXX}}(0.5, 0.5)$ (Dashed line).

RESULTS AND DISCUSSION

The following values of constants are considered as: $L = 1, \alpha = 1.11, a_1 = 2, a_2 = 1$, $\beta = 0.1$, $\omega = 0.1, \lambda = 0.1$ and we choose the proper value of the convergence control parameter $c_0 = -0.8$ to obtain convergent series solution. We have considered first 20 terms of series solution. Hence it gives an approximate solution of fingero-imbibition phenomenon in heterogeneous porous media with magnetic field effect. Table 1 indicates the numerical values of saturation of injected water for fingero-imbibition phenomenon at distance X for time T. Graphical presentation of the saturation of injected water is obtained by using Mathematica software. The graph of saturation of injected water versus distance X for fixed time $T = 0.1, 0.2, \ldots, 1$ is given in fig. 5 and fig. 6 represents the graph of saturation of injected water versus time T for fixed distance $X = 0.1, 0.2, \ldots, 1$

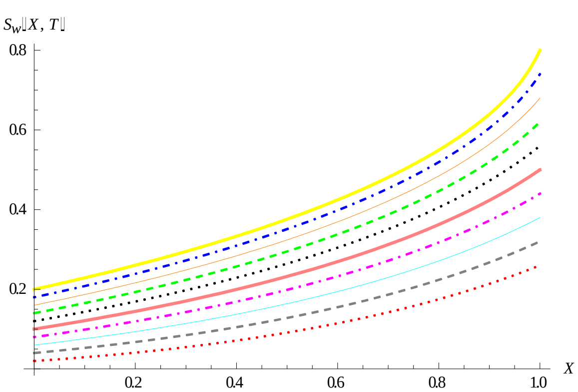

Figure 5: Saturation of water v/s distance X for fixed time $T = 0.1$(*lower most graph*), $0.2, \ldots, 1$(*upper most graph*).

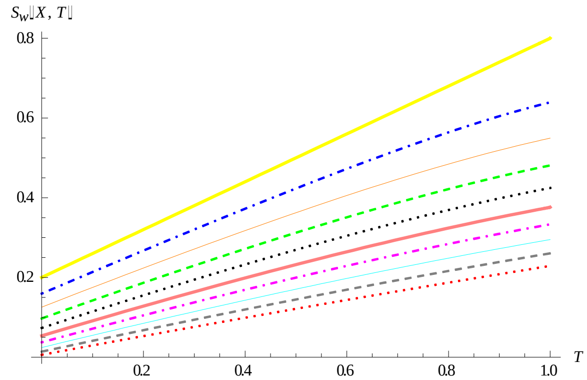

Figure 6: Saturation of water v/s time T for fixed distance $X = 0.1$(*lower most graph*), $0.2, \ldots, 1$(*upper most graph*).

COMPARATIVE STUDY WITH FINGERO-IMBIBITION PHENOMENON IN HETEROGENEOUS POROUS MEDIUM WITHOUT MAGNETIC FIELD EFFECT

Patel and Desai [13] have discussed homotopy series solution for fingero-imbibition phenomenon in heterogeneous porous medium without magnetic field effect. Table 2 shows the comparative numerical values of the saturation of injected water of fingero-imbibition phenomenon without magnetic field effect [13] and with magnetic field effect.

CONCLUSION

We have discussed the fingero-imbibition phenomenon in heterogeneous porous medium with magnetic field effect under certain assumptions. The homotopy series solution (32) represents the saturation of injected water. The solution (32) satisfies the boundary conditions (18). Numerical and graphical representations are obtained using Mathematica software. Table 1 indicates the numerical values of the saturation of injected water. Figures 5 and 6 give graphical representation. It is concluded that the saturation of injected water of fingero-imbibition phenomenon increases when the distance increases for given time T. Due to additional magnetic field effect the saturation of injected water is more increasing than the saturation of injected water without magnetic field effect. We can conclude that the magnetic field effect plays an important role in the fingero-imbibition phenomenon in the heterogeneous porous medium.

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Table-1: Numerical values of the saturation of injected water with magnetic field effect.

T	$X=0.1$	$X=0.2$	$X=0.3$	$X=0.4$	$X=0.5$	$X=0.6$	$X=0.7$	$X=0.8$	$X=0.9$	$X=1.0$
0.1	0.0294702	0.0409648	0.0548286	0.0714324	0.0911921	0.1145950	0.1422402	0.1749026	0.2136411	0.2600000
0.2	0.0528328	0.0676626	0.0848556	0.1048177	0.1280162	0.1550146	0.1865255	0.2235014	0.2672979	0.3200000
0.3	0.0759336	0.0938712	0.1141989	0.1373570	0.1638673	0.1943772	0.2297319	0.2711031	0.3202373	0.3800000
0.4	0.0987564	0.1195543	0.1427983	0.1689628	0.1986268	0.2325306	0.2716734	0.3174967	0.3722661	0.4400000
0.5	0.1212845	0.1446740	0.1705897	0.1995398	0.2321627	0.2692999	0.3121271	0.3624155	0.4231194	0.5000000
0.6	0.1435009	0.1691909	0.1975056	0.2289855	0.2643291	0.3044852	0.3508272	0.4055203	0.4724262	0.5600000
0.7	0.1653884	0.1930648	0.2234759	0.2571912	0.2949670	0.3378610	0.3874590	0.4463816	0.5196575	0.6200000
0.8	0.1869300	0.2162553	0.2484289	0.2840434	0.3239060	0.3691771	0.4216559	0.4844582	0.5640480	0.6800000
0.9	0.2081090	0.2387224	0.2722926	0.3094260	0.3509672	0.3981617	0.4529997	0.5190818	0.6044820	0.7400000
1.0	0.2289092	0.2604273	0.2949965	0.3332228	0.3759679	0.4245282	0.4810286	0.5494552	0.6393414	0.8000000

Table-2: Comparative numerical values of the saturation of injected water without magnetic field effect and with magnetic field effect.

T	$X=0.1$	$X=0.2$	$X=0.3$	$X=0.4$	$X=0.5$	$X=0.6$	$X=0.7$	$X=0.8$	$X=0.9$	$X=1.0$
0.1	0.0294301	0.0408938	0.0547358	0.0713267	0.0910823	0.1144897	0.1421481	0.1748322	0.2136012	0.2600000
0.1	0.0294702	0.0409648	0.0548286	0.0714324	0.0911921	0.1145950	0.1422402	0.1749026	0.2136411	0.2600000
0.2	0.0527940	0.0675937	0.0847656	0.1047151	0.1279096	0.1549123	0.1864359	0.2234330	0.2672591	0.3200000
0.2	0.0528328	0.0676626	0.0848556	0.1048177	0.1280162	0.1550146	0.1865255	0.2235014	0.2672979	0.3200000
0.3	0.0758961	0.0938046	0.1141118	0.1372578	0.1637642	0.1942783	0.2296453	0.2710369	0.3201998	0.3800000
0.3	0.0759336	0.0938712	0.1141989	0.1373570	0.1638673	0.1943772	0.2297319	0.2711031	0.3202373	0.3800000
0.4	0.0987202	0.1194901	0.1427143	0.1688671	0.1985275	0.2324355	0.2715902	0.3174333	0.3722302	0.4400000
0.4	0.0987564	0.1195543	0.1427983	0.1689628	0.1986268	0.2325306	0.2716734	0.3174967	0.3722661	0.4400000
0.5	0.1212496	0.1446122	0.1705089	0.1994478	0.2320674	0.2692088	0.3120477	0.3623551	0.4230855	0.5000000
0.5	0.1212845	0.1446740	0.1705897	0.1995398	0.2321627	0.2692999	0.3121271	0.3624155	0.4231194	0.5000000
0.6	0.1434674	0.1691316	0.1974281	0.2288975	0.2642381	0.3043985	0.3507520	0.4054637	0.4723946	0.5600000
0.6	0.1435009	0.1691909	0.1975056	0.2289855	0.2643291	0.3044852	0.3508272	0.4055203	0.4724262	0.5600000
0.7	0.1653563	0.1930080	0.2234018	0.2571073	0.2948806	0.3377792	0.3873886	0.4463291	0.5196290	0.6200000
0.7	0.1653884	0.1930648	0.2234759	0.2571912	0.2949670	0.3378610	0.3874590	0.4463816	0.5196575	0.6200000
0.8	0.1868992	0.2162010	0.2483583	0.2839637	0.3238244	0.3691004	0.4215907	0.4844107	0.5640233	0.6800000
0.8	0.1869300	0.2162553	0.2484289	0.2840434	0.3239060	0.3691771	0.4216559	0.4844582	0.5640480	0.6800000
0.9	0.2080794	0.2386705	0.2722254	0.3093505	0.3508905	0.3980903	0.4529401	0.5190399	0.6044621	0.7400000
0.9	0.2081090	0.2387224	0.2722926	0.3094260	0.3509672	0.3981617	0.4529997	0.5190818	0.6044820	0.7400000
1.0	0.2288809	0.2603777	0.2949326	0.3331515	0.3758960	0.4244623	0.4809749	0.5494194	0.6393273	0.8000000
1.0	0.2289092	0.2604273	0.2949965	0.3332228	0.3759679	0.4245282	0.4810286	0.5494552	0.6393414	0.8000000