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INVENTORY MODEL FOR THE RAYLEIGH DISTRIBUTED DETERIORATION UNDER DEMAND INCLINING MARKET CONDITION

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ABSTRACT

In this paper, we introduce a deterministic inventory model under replenishment policy over a fixed planning period for the deteriorating items. Demand functionfollows positive linear trend. The holding cost is considered to be constant and shortages are allowed and moderately backlogged. Further, we assume that the deterioration rate follows the Rayleigh distribution. The model is explained analytically by diminishing the total inventory cost and obtained optimum period.

KEY WORDS: Inventory Model, Deterioration, Rayleigh Distribution, Partial Backlogging, Inclining Demand

INTRODUCTION

In last few years many researchers have developed inventory models for perishable items such as medicines, electronic components, food items, drugs and fashion goods. As we know, in reality the inventory is continuously reducing due to its demand and deterioration. Moreover the demand is changes consistently over the time period, most of the research is carried out related to inventory models, with linearly changing demand. Furhter, researchers extended these inventory models by considering the time and price sensitivity. Silver and Meal [19] gave modified EOQ models for time varying demand pattern. Inventory replenishment policy for the linear trend in demand over a finite time horizon was analytically solved by Donaldson [3]. However, this model is much complex as computationally point of view. For more clarity Ritchie ([15], [16] and [17]) derived simple procedure to find exact solution of that model. Mitra etal. [12] formulated a simple procedure for adjusting the economic order quantity model for the case of both increasing / decreasing linear trend in demand.

Deterioration takes place in many real life situations such as an expire date of medicines, failure of the batteries as they age and spoilages of an items .Deterioration is either constant over time or the Weibull distributed (i.e. time dependent). Dave and Patel [2] derived an inventory model for deteriorating items with time dependent demand which is linearly propostional. Hollier and Mak [6] formulated ordering policies for deteriorating items where demand function deplates exponentially over time. Several authors Raafat [14], Shah and Shah [18] and Goyal and Giri [4] have narrated idea of deteriorating demand and developed the inventory models.

All the above discussed inventory models were formulated in a static environment where the demand is assumed to be constant and steady over a finite planning horizon. However, in the realistic product lifecycle, demand is increasing with time during the growth phase. Mandal [9] studied an EOQ model for the Weibull distributed deteriorating items under ramp-type demand and shortages. Mishraand Singh ([10], [11]) constructed an inventory model for ramp-type demand, time dependent deteriorating items with salvage value and shortages and deteriorating inventory model for time dependent demand and holding cost and with partial backlogging. Hung [7] investigated an inventory model with generalized type demand, deterioration and back order rates. Mishra et al. [13] considered time dependent demand and developed an inventory model for deteriorating items where deterioration rate and holding cost are constants shortages are allowed and partially backlogged.

Lord Rayleigh (1880) introduced the Rayleigh distribution in connection with problem in the field of acoustics. It has some relation with the Weibull, chi-square or extreme value distributions. The important characteristic of the Rayleigh distribution is that its hazard function is increase function of time. For more details, one can refer the Johnson, Kotz and Balakrishnant [8].

Recently, Acharya and Debata [1] introduced an inventory model for constant deteriorating with time increasing demand under partial backlogging.On the line of Acharya and Debata [1] we generalize the inventory models. Here, we consider linearly time dependent demand and develop an inventory model for deteriorating items have one parameter the Rayleigh distribution, in which shortages is allowed and partially backlogged. This paper is organized as follow: In Section 2, we give notation and assumptions. The probability density function, cumulative distribution function and hazard rate function are discussed in Section 3. The mathematical model for inventory is developed in Section 4 and concluding remarks are given in Section 5.

NOTATION AND ASSUMPTION

Fundamental notation and assumption used in this paper is as follows

Assumption:

- Demand is time dependent linear function
- The replenishment rate is infinite
- Deterioration rate follow one parameter Rayleigh distribution
- Shortages are allowed and are partially backlogged
- Lead time is zero

Notations:

- Demand rate $D(t) = a + bt;\ a, b > 0$ are constants
- $I(t)$ level of inventory at time t, $0 \leq t \leq T$
- T length of the cycle
- Deterioration rate $\theta(t) = \frac{t}{\sigma^2};\ \sigma > 0$ is scale parameter
- t_1 time when the inventory level reaches zero
- t_1^* optimal time
- A static ordering cost per order
- C_θ cost of deterioration per item
- C_h inventory holding cost per unit per unit time
- C_s shortage cost per unit per unit time
- I_m maximum inventory level for the ordering cycle, such that $I_m = I(0)$
- $C_T(t_1)$ average total cost per unit time under the condition $t_1 \leq T$

PROBABILITY DENSITY FUNCTION, CUMULATIVE DISTRIBUTION FUNCTION AND HAZARD RATE FUNCTION OF THE RAYLEIGH DISTRIBUTION

The random variable t is said to have one parameter Rayleigh distribution if its probability density function is given by

$$f(t;\sigma^2) = \frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}}\ ;\ t > 0\ ;\ \sigma > 0\ ,$$

where σ is scale parameter

and the corresponding cumulative distribution function is given by

$$F(t;\sigma^2) = \left(1 - e^{-\frac{t^2}{2\sigma^2}}\right);\ t > 0\ ;\ \sigma > 0\ ,$$

and its hazard (Deterioration) rate function is

$$\theta(t;\sigma^2) = \frac{\mathrm{f(t)}}{1-\mathrm{F(t)}} = \frac{t}{\sigma^2};\quad t > 0\ ;\ \sigma > 0$$

MATHEMATICAL MODEL

The behavior of the inventory system can be defined by the following differential equations:

$$\frac{dI(t)}{dt} = -D(t) - \theta(t)I(t)\ ;\quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -D(t)t_1 \leq t \leq T \quad (2)$$

With initial condition $I_m = I(0), I(t_1) = 0$

The solution of equation (1) and (2) using series expansion and neglecting the higher terms of σ^2 by considering $\sigma > 0.7$with boundary conditions is as follows

$$I(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{a}{6\sigma^2}(t_1^3 - t^3) +$$

$$\frac{b}{8\sigma^2}(t_1^4 - t^4) - \frac{a}{2\sigma^2}(t_1 - t)t^2 -$$

$$\frac{b}{2\sigma^2}(t_1^2 - t^2)t^2\ ;\ 0 \le t \le t_1 \quad (3)$$

$$I(t) = a(t_1 - T) + \frac{b}{2}(t_1^2 - T^2); \quad t_1 \le t \le T \quad (4)$$

Maximum inventory level can be computed as

$$I_m = I(0) = at_1 + \frac{bt_1^2}{2} + \frac{a}{6\sigma^2}t_1^3 + \frac{b}{8\sigma^2}t_1^4 \quad (5)$$

Now, total number of deteriorating items, sayD_T during time interval $[0, t_1]$ is

$$D_T = I_m - \int_0^{t_1} D(t)dt = \frac{a}{6\sigma^2}t_1^3 + \frac{b}{8\sigma^2}t_1^4 \quad (6)$$

Total number of inventory holding, sayH_T during the interval $[0, t_1]$is

$$H_T = \int_0^{t_1} I(t)dt = \frac{at_1^2}{2} + \frac{b}{3}t_1^3 + \frac{a}{12\sigma^2}t_1^4 + \frac{b}{30\sigma^2}t_1^5 \quad (7)$$

Total shortage quantity, say B_T during the interval $[t_1, T]$ is

$$B_T = -\int_{t_1}^{T} I(t)dt = \frac{a}{2}\left(T^2 + \frac{3}{2}t_1^2 - 2t_1T\right) + \frac{b}{6}(T^3 + 2t_1^3 - 3t_1^2T) \quad (8)$$

Hence, average cost per unit time under the condition $t_1 \le T$ is

$$C_T(t_1) = \frac{1}{T}[A + C_\theta D_T + C_h H_T + C_s B_T] \quad (9)$$

To minimized the total average cost $C_T(t_1)$ the necessary condition is $\frac{dC_T(t_1)}{dt_1} = 0$.

Which gives

$$\frac{1}{T}\Bigg[C_\theta\left(\frac{a}{2\sigma^2}t_1^2 + \frac{b}{2\sigma^2}t_1^3\right) + C_h\left(at_1 + bt_1^2 + \frac{a}{3\sigma^2}t_1^3 + \frac{b}{6\sigma^2}t_1^4\right) + C_s\left(\frac{3at_1}{2} - aT + bt_1^2 - bt_1T\right)\Bigg]$$

$$= g(t_1) \text{ (say)} \quad (10)$$

Now, $g(0) = -aC_s < 0$ and

$$g(T) = \frac{1}{T}\Bigg[C_\theta\left(\frac{a}{2\sigma^2}T^2 + \frac{b}{2\sigma^2}T^3\right) + C_s\left(\frac{aT}{2}\right) + C_h\left(aT + bT^2 + \frac{a}{3\sigma^2}T^3 + \frac{b}{6\sigma^2}T^4\right)\Bigg] > 0$$

and

$$g'(t) = \frac{1}{T}\Bigg[C_\theta\left(\frac{at_1}{2\sigma^2} + \frac{3b}{2\sigma^2}t_1^3\right) + C_h\left(a + 2bt_1 + \frac{a}{\sigma^2}t_1^2 + \frac{2b}{3\sigma^2}t_1^3\right) + C_s\left(\frac{3a}{2} + 2bt_1 - bT\right)\Bigg] > 0$$

Which indicates that the function is strictly monotonically increasing function and equation (10) has unique solution (say) t_1^* where$t_1^* \epsilon [0, T]$ which gives the minmum answer.
Therefore, we have

Property:
Inventory model for the rayleigh distributed deterioration under the condition $0 < t_1 < T$, $C_T(t_1)$ obtains its minimumat $t_1 = t_1^*$ where $g(t_1^*) = 0$ if $t_1^* < T$.

CONCLUDING REMARKS

In this paper, we studied replenishment policy over a fixed planning period for a deteriorating items having deterministic demand with positive linear trend and shortages. The holding cost was considered to be constant over a planning period and the deteriorating items follow the Rayleigh distribution. The model was solved analytically by minimizing the total inventory cost and obtained optimal period. In future, one can generalized the model by applying time varying holding cost and by considering time varying demand having Weibull distribution. Numerical analysis can be performed to validate solution obtained for inventory policy.

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