

was analyzed by Andharia et al. [10]. Patir and Cheng [14-15] modified the averaged Reynolds' equation for rough surfaces. They defined pressure and shear flow factors which were obtained independently by numerical flow simulation using randomly generated roughness profiles. Etsion and Izhak [16] analyzed that theoretical modelling of surface texturing in hydrodynamic lubrication is a necessary first step to obtain favorable effect of the texturing. The effects of surface roughness characteristics on the load carrying capacity of tilt pad thrust bearings with water lubrication were studied through the average flow model by Wang, Yuechang et al. [18].

In this paper, an analysis has been conducted to evaluate the influence of surface roughness parameters and flow factors which are strongly dependent on the surface pattern parameter on a transversely rough plane inclined slider bearing.

ANALYSIS

Patir and Cheng [14-15] developed "Averaged Reynolds' equation" which took an account of the surface topography [17] (Fig. 1).

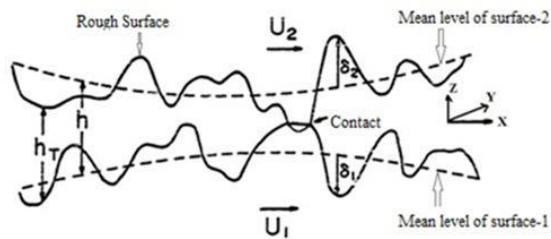


Fig. 1 Surface roughness and film geometry

The mean pressure in a rough slider bearing is governed by the averaged Reynolds' equation:

$$\frac{\partial}{\partial x} \left[\varphi_x \frac{h^3}{12\mu} \frac{\partial \bar{p}}{\partial x} \right] + \frac{\partial}{\partial y} \left[\varphi_y \frac{h^3}{12\mu} \frac{\partial \bar{p}}{\partial y} \right] \quad (1)$$

$$= \frac{U}{2} \frac{\partial \bar{h}_T}{\partial x} + \frac{U\sigma}{2} \frac{\partial \varphi_s}{\partial x}$$

where φ_x and φ_y are pressure flow factors, φ_s is shear flow factor, $h_T = h + \delta$ is local film thickness, h is nominal film thickness

(compliance), \bar{h}_T is expected value of mean gap between two surfaces (separation), U is velocity of slider, μ is viscosity of lubricant, $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$ is composite r.m.s. roughness and \bar{p} is mean pressure level.

The investigation assumes that the flow of lubricant is steady and in X-direction only. Moreover, for transversely rough surface ($\gamma < 1$), the variations in roughness heights in X-direction are significant [17] (Fig. 2) so far as the performance is concerned. Hence, the effect of shear flow factor- φ_s is significant, which of course is negligible in the case of longitudinal rough surface ($\gamma > 1$). Here γ is ratio of x and y correlation lengths of roughness.

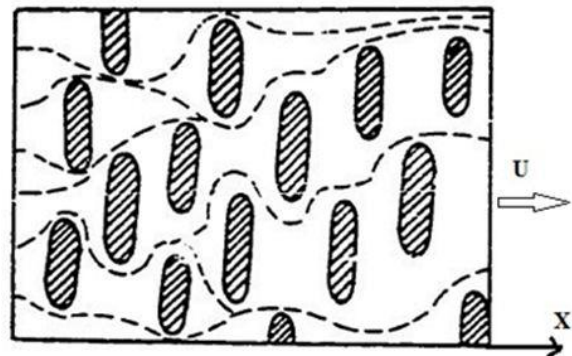


Fig. 2 Transverse rough surface ($\gamma < 1$)

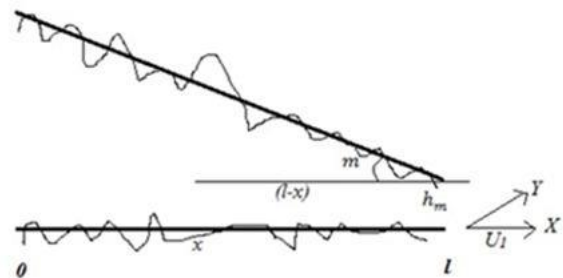


Fig. 3 Inclined plane slider bearing geometry

In view of above assumptions, Eq. (1) results in,

$$\frac{\partial}{\partial x} \left[\varphi_x \frac{h^3}{12\mu} \frac{\partial \bar{p}}{\partial x} \right] = \frac{U}{2} \frac{\partial \bar{h}_T}{\partial x} + \frac{U\sigma}{2} \frac{\partial \varphi_s}{\partial x} \quad (2)$$

For a rough plane slider bearing having the configuration as shown in Fig. 3, one considers

$$h = h_m + m(l - x) \quad (3)$$

where, h_m is minimum film thickness at the trailing edge of the slider bearing, l is the length of the slider bearing, m is the inclination of the slider bearing (Fig. 3).

Since δ is assumed to be stochastic in nature and is governed by the probability density function $f(\delta)$, $-c < \delta < c$, the mean α , the standard deviation σ and the skewness parameter ε are described as in Christensen – Tonder [4-7] and Deheri et al. [8] as:

$$E(R) = \int_{-c}^c R f(\delta) d\delta \quad (4)$$

$$E(\delta) = \alpha \quad (5)$$

$$E[(\delta - \alpha)^2] = \sigma^2 \quad (6)$$

And

$$E[(\delta - \alpha)^3] = \varepsilon \quad (7)$$

Where, c is maximum deviation from the mean film thickness. It is noteworthy that while α and ε can assume both positive and negative values, σ is always positive.

Following the discussion of in Christensen - Tonder [4-7], an approximation to $f(\delta)$ is,

$$f(\delta) = \begin{cases} \frac{32}{35c} \left(1 - \frac{\delta^2}{c^2} \right) & -c \leq \delta \leq c \\ 0 & \text{elsewhere} \end{cases} \quad (8)$$

Thus \bar{h}_T can be approximated as,

$$\bar{h}_T = \frac{13h}{8} \approx h \quad (9)$$

Now as per the averaging process discussed by Deheri et al. [8], Eq. (2) reduces to,

$$\frac{d}{dx} \left[\varphi_x \frac{g(h)}{12\mu} \frac{d(\bar{p})}{dx} \right] = \frac{U}{2} \frac{dh}{dx} + \frac{U\sigma}{2} \frac{d\varphi_s}{dx} \quad (10)$$

Where (\bar{p}) is expected value of the mean pressure level \bar{p} and

$$g(h) = h^3 + 3h^2\alpha + 3h(\sigma^2 + \alpha^2) + (\varepsilon + 3\sigma^2\alpha + \alpha^3) \quad (11)$$

The empirical relations for φ_x and φ_s provided by Patir [17] are as under:

$$\varphi_x = 1 - C e^{-rH} \quad (\gamma \pi 1) \quad (12)$$

And

$$\varphi_s = A_1 H^{\alpha_1} e^{-\alpha_2 H + \alpha_3 H^2} \quad (\gamma \pi 1) \quad (13)$$

The dimensionless forms of these flow factors are

$$\varphi_x = 1 - C e^{-r h^* H_m} \quad (\gamma \pi 1) \quad (14)$$

And

$$\varphi_s = A_1 (h^* H_m)^{\alpha_1} e^{-\alpha_2 (h^* H_m) + \alpha_3 (h^* H_m)^2} \quad (\gamma \pi 1) \quad (15)$$

Where

$$H = \frac{h}{\sigma}, h^* = \frac{h}{h_m}, H_m = \frac{h_m}{\sigma} \quad (16)$$

While the constants C , r , A_1 , α_1 , α_2 and α_3 are given as functions of γ in the Table 1 and Table 2 (Patir [17]).

Making use of the following dimensionless quantities

$$h^* = \frac{h}{h_m}, X = \frac{x}{l}, m^* = \frac{ml}{h_m}, \bar{P} = \frac{h_m^2 (\bar{p})}{\mu U l}, \alpha^* = \frac{\alpha}{h_m}, \sigma^* = \frac{\sigma}{h_m}, \varepsilon^* = \frac{\varepsilon}{h_m^3} \quad (17)$$

And

$$G(h^*) = h^{*3} + 3h^{*2}\alpha^* + 3h^*(\sigma^{*2} + \alpha^{*2}) + (\varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3}) \quad (18)$$

Eq. (10) turns to the dimensionless form

$$\frac{d}{dx} \left[\varphi_x G(h^*) \frac{d\bar{P}}{dX} \right] = 6 \frac{d}{dX} [h^* + \sigma^* \varphi_s] \quad (19)$$

With the aid of the following boundary conditions:

$$\bar{P} = 0, \text{ at } X = 0 \text{ and } 1 \quad (20)$$

$$\frac{d\bar{P}}{dX} = 0, \text{ at which the mean gap is}$$

maximum

Eq. (19) leads to,

$$\bar{P}(X) = \int_0^X \frac{6(h^* + \sigma^* \varphi_X) - Q^*}{\varphi_X G(h^*)} dX \quad (21)$$

Where,

$$Q^* = \frac{\int_0^1 \frac{6(h^* + \sigma^* \varphi_X)}{\varphi_X G(h^*)} dX}{\int_0^1 \frac{1}{\varphi_X G(h^*)} dX} \quad (22)$$

The dimensionless load carrying capacity per unit width is given by,

$$W^* = \frac{wh_m^2}{\mu Ul} = \int_0^1 \bar{P} dX$$

$$\therefore W^* = \int_0^1 \left[\int_0^X \frac{6(h^* + \sigma^* \varphi_X) - Q^*}{\varphi_X G(h^*)} dX \right] dX \quad (23)$$

RESULTS AND DISCUSSION

In Fig. 4-6, we have the variations in load carrying capacity of the bearing with respect to α , which establish that the load carrying capacity increases with the increasing values of α (-ve) while α (+ve) causes decrease in the load carrying capacity, a property similar to the case of longitudinal roughness pattern [12,13]. We can observe that the increasing values of standard deviation σ , skewness ε (+ve) and roughness pattern parameter γ decrease the load carrying capacity.

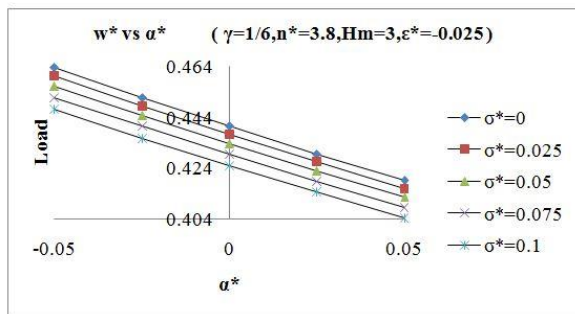


Fig. 4 Variation of load carrying capacity with respect to α^*

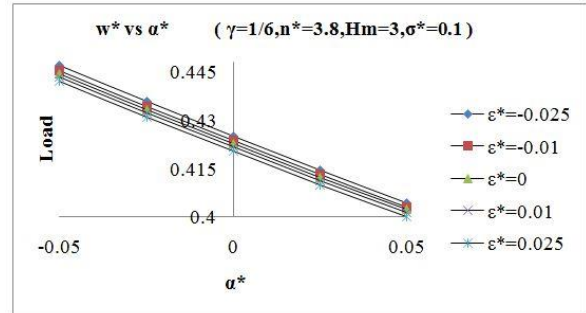


Fig. 5 Variation of load carrying capacity with respect to α^*

The variations in load carrying capacity of the bearing with respect to the standard deviation σ can be seen from Fig. 7-9, for various values of ε , α and γ . These reveal that the load carrying capacity enhances due to the decreasing values of standard deviation, increasing values of negatively skewed roughness, increasing values of variance (-ve) and decreasing values of roughness pattern parameter γ of the surfaces.

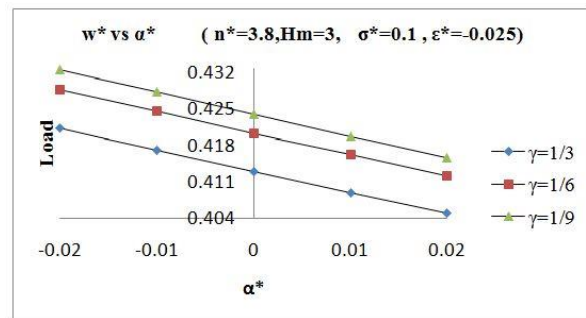


Fig. 6 Variation of load carrying capacity with respect to α^*

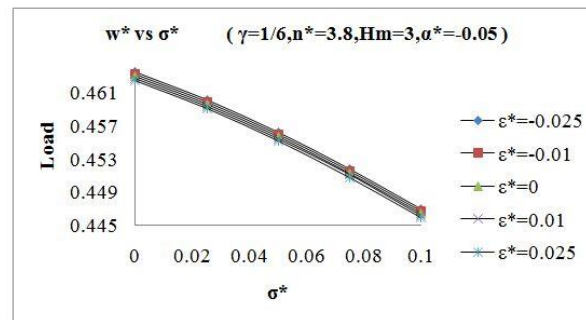


Fig. 7 Variation of load carrying capacity with respect to σ^*

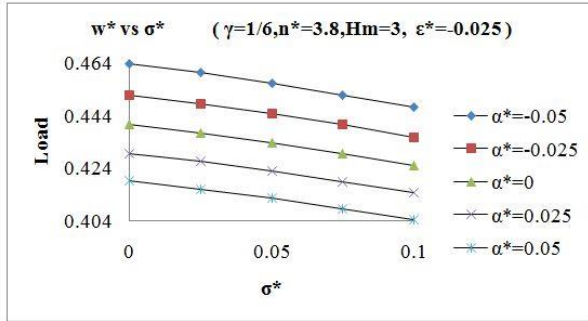


Fig. 8 Variation of load carrying capacity with respect to σ^*

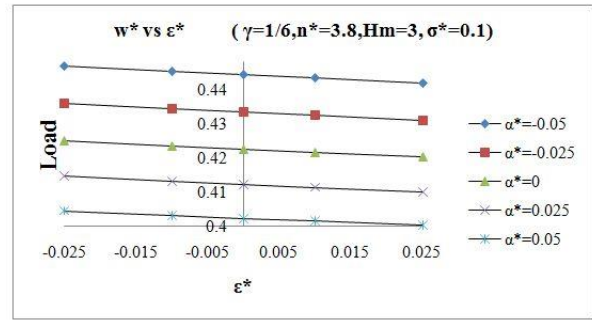


Fig. 11 Variation of load carrying capacity with respect to ϵ^*

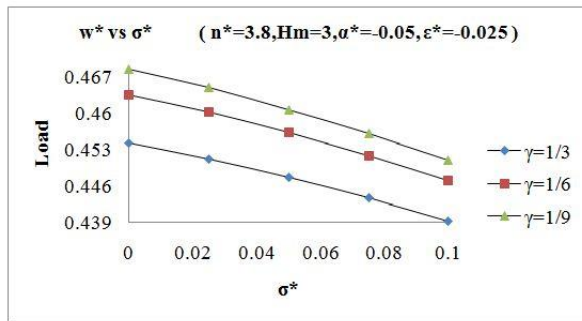


Fig. 9 Variation of load carrying capacity with respect to σ^*

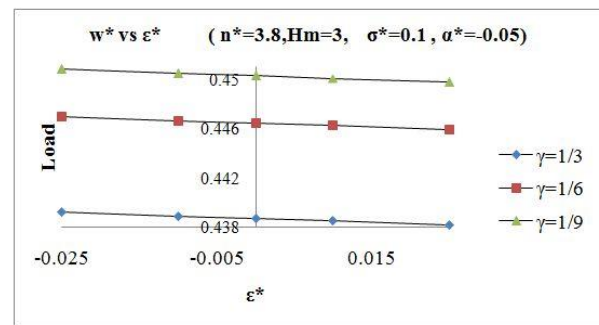


Fig. 12 Variation of load carrying capacity with respect to ϵ^*

Fig. 10-12 dealing with the effect of skewness on the load carrying capacity establishes that the load carrying capacity increases with the negatively skewed roughness, similar to the case of longitudinal roughness pattern [12-13] but the rate at which it goes up is very small.

The trend of load carrying capacity with respect to the roughness pattern parameter γ amplifies as the surface is more transverse while in the case of longitudinal roughness [12] the load carrying capacity reduces as the surface roughness is more longitudinal.

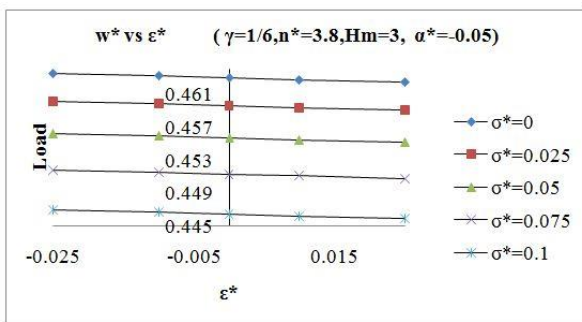


Fig. 10 Variation of load carrying capacity with respect to ϵ^*

CONCLUSION

In this paper, the effect of roughness parameters on the pressure and load carrying capacity for a rough finite inclined plane slider bearing with transversely rough surface has been analyzed. The results obtained here are compared with those of [8, 12-13]. It is observed that the load carrying capacity can be increased by decreasing the value of the roughness pattern parameter. This investigation suggests that the adverse effect of transverse surface roughness can be overcome to a great extent in the case of negatively skewed roughness under the presence of variance (-ve). However, from longevity point of view, the roughness aspect needs to be evaluated while designing the bearing system.

REFERENCES

- [1] Tonder, K. C. (1972): Surface distributed waviness and roughness. First world conference in Industrial Tribology. Vol. 3.
- [2] Ke, Liao-Liang, et al. (2010): Sliding frictional contact analysis of functionally graded piezoelectric layered half-plane. Acta mechanica 209.3-4: 249-268.

- [3] Thomas, Evan, Mircea D. Pascovici, and Romeo P. Glovnea (2015): Load carrying capacity of a heterogeneous surface bearing. *Friction* 3.4: 287-293.
- [4] Christensen H. (1969): Stochastic Models for Hydrodynamic Lubrication of Rough Surfaces. *Proceedings of the Institution of Mechanical Engineers*, vol. 184 no. 1 1013-1026.
- [5] Christensen H., Tonder K. (1971): The Hydrodynamic Lubrication of Rough Bearing Surfaces of Finite Width. *Journal of Lubrication Tech* 93(3), 324-329.
- [6] Christensen H., Tonder K. (1972): Waviness and Roughness in Hydrodynamic Lubrication. *Proceedings of the Institution of Mechanical Engineers*, vol. 186 no. 1, 807-812.
- [7] Christensen H. (1972): A Theory of Mixed Lubrication. *Proceedings of the Institution of Mechanical Engineers*, vol. 186 no. 1 421-430.
- [8] Deheri, G. M., P. I. Andharia, and R. M. Patel. (2005): Transversely rough slider bearings with squeeze film formed by a magnetic fluid. *International Journal of Applied Mechanics and Engineering* 10.1: 53-76.
- [9] Chiang, Hsiu-Lu, et al. (2005): Surface roughness effects on the dynamic characteristics of finite slider bearings. *Journal of CCIT* 34.1: 1-11.
- [10] P. I. Andharia, J. L. Gupta and G. M. Deheri. (2001): Effect of Surface Roughness on Hydrodynamic Lubrication of Slider Bearings. *Tribology Transactions*, Volume 44, Issue 2, pages 291-297.
- [11] Dersse, Getachew A., and Prawal Sinha. (2011): THD analysis for finite slider bearing with roughness: special reference to load generation in parallel sliders. *Acta mechanica* 222.1-2: 1-15.
- [12] Panchal Girishkumar C., Himanshu C. Patel and G. M. Deheri. (2016): Influence of surface roughness through a series of flow factors on the performance of a longitudinally rough finite slider bearing. *Annals of the Faculty of Engineering Hunedoara-International Journal of Engineering* 14.2.
- [13] Panchal, G. C., H. C. Patel, and G. M. Deheri. (2016): Influence of magnetic fluid through a series of flow factors on the performance of a longitudinally rough finite slider bearing. *Global Journal of Pure and Applied Mathematics* 12.1: 783-796.
- [14] Patir, Nadir, and H. S. Cheng. (1978): An average flow model for determining effects of three-dimensional roughness on partial hydrodynamic lubrication. *Journal of Tribology* 100.1: 12-17.
- [15] Patir, Nadir and H. S. Cheng. (1979): Application of average flow model to lubrication between rough sliding surfaces. *Journal of Tribology* 101.2: 220-229.
- [16] Etsion, Izhak. (2013): Modeling of surface texturing in hydrodynamic lubrication. *Friction* 1.3: 195-209.
- [17] Patir N (1978): Effects of surface roughness on Partial film lubrication using an average flow model based on numerical simulation. Northwestern University, Ph.D. Chapter-2.
- [18] Wang, Yuechang, et al. (2017): Surface roughness characteristics effects on fluid load capability of tilt pad thrust bearings with water lubrication. *Friction*: 1-10.

Table-1: Relation between γ , C, r and H

γ	C	r	H
1/3	1.16	0.42	$H > 0.75 > 0.5$
1/6	1.38	0.42	$H > 1 > 0.5$
1/9	1.48	0.42	$H > 1 > 0.5$

Table-2: Relation between γ , A₁, α_1 , α_2 , α_3 and H

γ	A ₁	α_1	α_2	α_3	H=h/ σ
1/3	1.858	1.01	0.76	0.03	$H > 0.5$
1/6	1.962	1.08	0.77	0.03	$H > 0.5$
1/9	2.046	1.12	0.78	0.03	$H > 0.5$