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SEMILEPTONIC DECAY OF B-MESON INTO D(D*) STATES IN A QUARK MODEL

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ABSTRACT

The exclusive semileptonic decay form factors and widths of B meson in the D(D*) states based on heavy quark effective theory are computed. The spectroscopic parameters of B, D and the D* states deduced using the phenomenological coulomb plus power form of the q -q potential (CPP_ν) with power exponent ν have been employed to compute the Isgur-Wise function as well as the decay form factors. The universal behaviour of the Isgur-Wise function at the maximum momentum transfer has been observed for different choices of ν between 0.5 to 2.0. However, such an universality is not observed for the choice of ν < 0.5. The semileptonic decay found to occur in the range of potential exponent $\nu \approx 0.7$ in the case of $B \to D l \bar{\nu}_l$ and $B \to D^* l \bar{\nu}_l$ are in agreement with the experimental values..

INTRODUCTION

Weak decays of heavy mesons have attracted the interest of theorists for a long time [1, 2, 3]. Either in semileptonic or in nonleptonic decays one will encounter the calculation of hadronic matrix elements and the treatment of hadronic matrix elements is one of the most essential tasks in studying heavy meson decays. The heavy quark effective theory (HQET) [4] may provide a powerful approach to study the transition between heavy quarks, but it may not properly deal with decays of heavy quarks into light flavours. Moreover, the calculation of transition matrix elements involves nonperturbative dynamics which is now difficult to calculate from first principles of QCD. At present the QCD motivated quark models may provide a good phenomenological description and are therefore very useful for the understanding of heavy meson decays.

Exclusive decays of heavy flavour hadrons play a complementary role in the determination of fundamental parameters of the electroweak standard model and in the development of a deeper understanding of QCD. Exclusive decays into few-body final states are often easier to measure, but the theory of exclusive processes is more demanding and hence still under developed. In view of the exciting experimental prospects at future bottom and charm factories, where many channels are expected to be measured accurately and hence, there is a pressing demand to make further progress on the theoretical side.

The semileptonic decays in exclusive modes are particularly important for testing the dynamics of heavy flavours. The form factors involved in the bottom decaying into charm in the present case are determined within the framework of heavy flavour symmetry (HFS), since both b and c are heavy. The form factors relevant to semileptonic and nonleptonic decays are related to the single function known as Isgur-Wise function ξ(ω) in heavy flavour symmetry. Since the Isgur-Wise function cannot be calculated from first principles, many models and non-perturbative approaches, which exhibit the heavy quark symmetry, have been employed to describe relevant phenomena. However, it was found that the finite mass corrections are very important [5]. It appears that in some sense a step back should be done from using the heavy quark symmetry as a guide under model building to the straightforward calculations with full quark propagators.

The study of semi-leptonic decays of heavy quarks provides the cleanest avenue for the determination of the Cabibbo-Kobayashi-Maskawa matrix elements, which are fundamental parameters in the standard model of particle physics. The coupling strength of the weak b → c transition is proportional to |Vcb|, which has been measured in exclusive semileptonic transitions, $B \to D l \bar{\nu}_l$ [6,7,8,9] and $B \to D^* l \bar{\nu}_l$ [6,9,10,11,12,13]. The charge of the detected lepton identifies the flavour of the B hadron according to the $\Delta B = \Delta Q$ rule. Also, the semileptonic branching ratios are large in B decays, allowing for extensive experimental investigations. These modes are used to measure the CKM matrix elements Vcb and Vub and the size of the $B^0 - \bar{B}^0$ mixing.

EXCLUSIVE SEMILEPTONIC DECAY FORM FACTORS

Theoretical predictions for semileptonic decays to exclusive final states require knowledge of the form factors, which parameterize the hadronic current as function of momentum transfer, $q^2 = (pB - pD)^2$ [14]. These decays are the simplest to understand theoretically through the spectator diagram as shown in the Fig. 1. Only the form factors are unknown, but they can be reliably determined from HFS. The semileptonic Bmeson decay amplitude is determined by the matrix elements ofthe weak Hamiltonian as [15],

$$H_W = \frac{G_F}{\sqrt{2}} V_{cb} [\bar{c}\gamma_\mu (1-\gamma_5) b][\bar{e}\gamma^\mu (1-\gamma^5)\nu_e] \qquad (1)$$

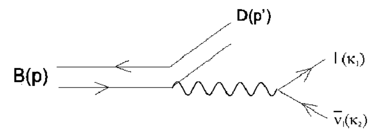

Fig 1: Feynmann diagram for semileptonic decays of B mesons into D mesons

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The matrix element factors into the product of the leptonic and the hadronic matrix elements. The hadronic part is the matrix element of the vector or axial vector current between Band D states. The vector and axial vector part of the matrix elements are given by,

$$V^{\mu} = \bar{c}\gamma_{\mu} b \qquad (2)$$

and

$$A^{\mu} = \bar{c}\gamma^{\mu}\gamma_5 b \qquad (3)$$

In the case of the final D states corresponds to the J = 0 states, as the matrix element of any axial current A_{μ} between the two pseudo-scalar mesons vanishes, only vector current V_{μ} contributes. Unlike in the case of electromagnetic current of the charged p ions, here the vector current $V^{\mu} = \bar{c}\gamma_{\mu} b$ is not conserved as $q_{\mu}V^{\mu} \propto (m_b - m_c) \neq 0$. So the matrix element of the hadronic current, V^{μ} between the two $J^P = 0^-$ mesons is expressed in terms of two form factors $f\pm(q^2)$ as,

$$\langle D(p')|V|B(p)\rangle = f_+(q^2)(p+p')^{\mu} + f_-(q^2)(p-p')^{\mu} \qquad (4)$$

Where $q = p - p' = k_1 + k_2$ is the four momentum transfer and $f+(q^2)$ and $f-(q^2)$are the dimensionless weak transition form factors corresponds to B⟶ D, which are functions of the invariant q^2. Here $q^2 = (k_1 + k_2)^2$ also represents the invariant mass of the lepton pair and it varies within the range, $m_l^2 \le q^2 \le (M_B - M_D)^2 = q^2_{max}$

The transition between the pseudo scalar B and the vector $D_q^{*}(p',\varepsilon)$ mesons depends on four independent form factors as [15],

$$\left\langle D^{*}(s)(p',\varepsilon)\left|\bar{c}\gamma_{\mu} b\right|B(p)\right\rangle = 2i\varepsilon^{\mu\nu\alpha\beta}\frac{\varepsilon_{\nu}p'_{\alpha}p_{\beta}}{M_B + M_{D^*}}V(q^2) \quad (5)$$

$$\left\langle D^{*}(s)(p',\varepsilon)\left|\bar{c}\gamma_{\mu}\gamma_5 b\right|B(p)\right\rangle = (M_B + M_{D^*})[\varepsilon^{\mu} - \frac{\varepsilon.q q^{\mu}}{q^2}]A_1(q^2) - \varepsilon.q[\frac{(p+p')^{\mu}}{M_B + M_{D^*}} - \frac{(M_B - M_{D^*})}{q^2}q^{\mu}]A_2(q^2) + 2M_{D^*}\frac{\varepsilon.q q^{\mu}}{q^2}A_0(q^2) \quad .....(6)$$

Where, ε is the polarization factor of vector meson. On the basis of HQET, the most general form of the transition discussed by Eqns. 4 and 5 can be expressed as [15],

$$\frac{1}{\sqrt{M_B M_D}}\left\langle D(v')\left|V^{\mu}\right|B(v)\right\rangle = (v + v')^{\mu}\xi(\omega) \qquad (7)$$

$$\frac{1}{\sqrt{M_B M_{D^*}}}\left\langle D^{*}(v',\varepsilon_3)\left|V^{\mu}\right|B(v)\right\rangle = i\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\nu}v'_{\alpha}v_{\beta}\xi(\omega) \qquad (8)$$

$$\frac{1}{\sqrt{M_B M_{D^*}}}\left\langle D^{*}(v',\varepsilon_3)\left|A^{\mu}\right|B(v)\right\rangle = [(1+\omega)\varepsilon^{\mu} - (\varepsilon\ .v\ )v'_{\mu}]\xi(\omega) \qquad (9)$$

where ξ(ω) is a universal function known as Isgur-Wise function and ω is given by,

$$\omega = v.v' = \frac{M^2_B + M_D^2 - q^2}{2M_B M_D} \qquad (10)$$

The Isgur-Wise function, ξ(ω) can be evaluated according to the relation given by [16],

$$\xi(\omega) = \frac{2}{\omega - 1}\left\langle j_0(2E_q\sqrt{\frac{\omega-1}{\omega+1}}r)\right\rangle \qquad (11)$$

where $E_q$is the binding energy of the decaying meson, which is obtained by solving the Schrodinger equation Hψ = Eψ and $j_0(2E_q\sqrt{\frac{\omega-1}{\omega+1}}r)$ represents the spherical Bessel function of order zero. The angular bracket, <A> corresponds to

$$\langle A\rangle = \int_0^{\infty} dr r^2 R_i(r)A(r)R_f(r) \qquad (12)$$

Consequently, the form factors $f_{\pm}(q^2)$ correspond to the D final state are related to the Isgur-Wise function as [15],

$$f_{\pm}(q^2) = \xi(\omega)\frac{M_B \pm M_D}{2\sqrt{M_B M_D}} \qquad (13)$$

and that related to the D*as the final hadronic state is given by [15]

$$V(q^2) = A_2(q^2) = [1 - \frac{q^2}{(M_B + M_{D^*})^2}]^{-1}A_1(q^2) = \frac{(M_B + M_{D^*})^2}{4M_B M_{D^*}}\xi(\omega) \quad (14)$$

It is evident from Eqn. 14 that in the limit, $q^2 \longrightarrow 0$

$$V(q^2) = A_2(q^2) = A_1(q^2) = A_0(q^2) \qquad (15)$$

Thus knowing the masses and wave functions of the initial and final states of the hadrons, we can compute the Isgur-Wise function ξ(ω) and the respective form factors correspond to $B \to D(D^*) + l^+\nu_l$ transitions.

DECAY RATES OF $B \to D(D^*) + l^+\nu_l$

With the help of these form factors one can readily compute the B_{ct3} decay rates according to the general scheme. Here, in the present study, we consider either electron or mu on as the leptons in the final state and m_{ℓ} is neglected. In this case the contribution from the form factor $f-(q^2)$ can also be neglected. The differential decay rates $\frac{d\Gamma}{dq^2}$ for $B \to D + l^+\nu_l$ then be expressed in terms of $f+(q^2)$ as [15],

$$\frac{d\Gamma}{dq^2}(B \to D + l^+\nu_l) = \frac{G^2_F\left|V_{cb}\right|^2\left|f_+\right|^2}{192\pi^3 M^3_B}[(q^2 - M^2_B - M_D^2)^2 - 4M_B^2 M_D^2]^{3/2} \quad (16)$$

Or in terms of ω

$$\frac{d\Gamma}{d\omega}(B \to D + l^+\nu_l) = G_F^2\left|V_{cb}\right|^2(M_B + M_D)^2 M_D^3\sqrt{\omega^2 - 1}\xi^2(\omega) \quad (17)$$

Similarly the decay rate for $B \to D^* + l^+\nu_l$ can be written as,

$$\frac{d\Gamma}{dq^2}(B \to D^* + l^+\nu_l) = \frac{G^2_F\left|V_{cb}\right|^2 q^2 M_{D^*}\sqrt{\omega^2 - 1}}{96\pi^3 M^3_B}(H_0{}^2 + H_+{}^2 + H_-{}^2) \quad (18)$$

Where, H_0 and $H_{\pm}$ are longitudinal and transverse helicities respectively and are related to the form factors as,

$$H_0(q^2) = \frac{M_B + M_{D^*}}{2M_{D^*}\sqrt{q^2}}[(M^2_B - M^2_{D^*} - q^2)A_1(q^2) - \frac{4M^2_B M^2_{D^*}(\omega^2 - 1)}{(M_B + M_{D^*})^2}A_2(q^2) \quad (19)$$

$$H_{\pm}(q^2) = (M_B + M_{D^*})[A_1(q^2) \mp \frac{2M_B M_{D^*}\sqrt{(\omega^2 - 1)}}{(M_B + M_{D^*})^2}V(q^2)] \quad (20)$$

With the help of Eqn. 16 and 18 we compute the decay widths for the channel by integrating over q^2.

THE PHENOMENOLOGY AND EXTRACTION OF THE SPECTROSCOPIC PARAMETERS

For the description of the meson bound states, we adopt the phenomenological Coulomb plus power potential (CPP*ν*) expressed as [17, 18],

$$V(r) = -\frac{4\alpha_s}{3r} + Ar^{\nu} \qquad (21)$$

Here, *A* is the confinement strength of the potential and as is the running strong coupling constant which is computed as,

$$\alpha_s(\mu^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f)\ln(\frac{\mu^2}{\Lambda^2})} \quad (22)$$

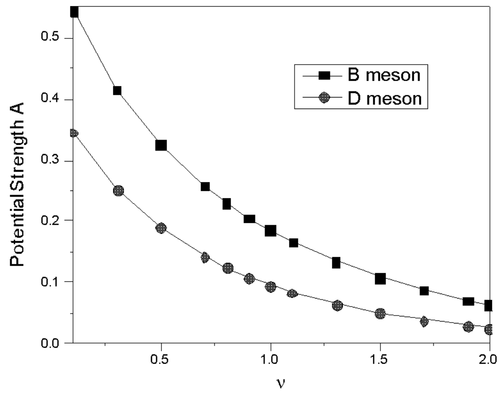

Fig 2: Potential strength A in $GeV^{\nu+1}$ obtained from the ground state spin-average mass against the choices of potential exponent, ν.

where, n_f is the number of flavours, m is the renormalization scale related to the constituent quark mass and L is the QCD scale which is taken as 0.150 GeV by fixing α_s= 0.118 at the Z-boson mass (91 GeV) [19].

The potential parameter, A of Eqn.21 is similar to the string strength σ of the Cornell potential. We particularly chose to vary ν in our study as very different inter-quark potentials can provide fairly good description of the mass spectra, while the transitions and other decay properties are very sensitive to the wave functions. And the wave functions vary differently with different choice of inter-quark potential. Thus the present study varies the exponent ν (0.1 ≤ ν ≤ 2.0). It can also provide significant understanding of the quark-antiquark interaction in the mesonic states while they undergo a transition or decay through annihilation channels. The different choices of ν here then correspond to different potential forms. So, the potential parameter A expressed in $GeV^{\nu+1}$ can be different for each choices of ν. The model potential parameter A and the mass parameter of the quark/antiquark (m_1,m_2) are fixed using the known ground state center of weight (spin average) mass and the hyperfine splitting ($M_{^3S_1} - M_{^1S_0}$) of the ground state D and B systems respectively. The spin average mass of the ground state is computed for the different choices of ν in the range, 0.1 ≤ ν ≤ 2.0.The spin average or the center of weight mass, MCW is calculated from the known experimental/theoretical values of the pseudoscalar (J = 0) and vector (J = 1) mesonic mass as,

$$M_{n,CW} = \frac{\sum_J (2J+1) M_{nJ}}{\sum_J (2J+1)} \quad (23)$$

The Schrödinger equation is numerically solved using the Mathematica notebook of the Runge-Kutta method [20]. For computing the mass difference between different spin degenerate mesonic states, we consider the spin dependent part of the usual one gluon exchange potential (OGEP) given by [21, 22, 23, 24, 25]. Accordingly, the spin dependent part, V_{SD}(r) for the angular quantum number ℓ = 0 contains only the spin-spin hyperfine interaction given by

$$V_{SD}(r) = V_{SS}(r)[S(S+1) - \frac{3}{2}] \quad (24)$$

The coefficient of these spin-dependent terms of Eqn.24 is given by the usual one gluon exchange (OGE) interaction as [23]

$$V_{SS}(r) = \frac{16\pi\alpha_s}{9 m_1 m_2}\delta^{(3)}(r) \quad (25)$$

The computed masses of the B and D states are listed in Table 1. The spectroscopic parameters thus correspond to the fitted quark masses, the potential strength A, the potential exponent, ν and the corresponding radial wave functions. The fitted mass parameters are m_c= 1.28 GeV/c^2, m_b= 4.4 GeV/c^2 and m_u= 0.336 GeV/c^2 while the potential strength A for each choices of ν are shown in Fig. 2. The numerical solution for the radial wave functions thus obtained for the different choices of the potential exponent, ν are plotted in Fig. 3 and Fig. 4 in the case of B and D systems respectively.

Table 1: Mass spectra of ground state B(B*) and D(D*) mesons in GeV

Potential Exponent ν	M_{B^*} GeV	M_B GeV	M_{D^*} GeV	M_D GeV
0.1	5.319	5.294	1.985	1.932
0.3	5.324	5.279	1.994	1.905
0.5	5.329	5.266	2.001	1.883
0.7	5.332	5.255	2.007	1.864
0.8	5.334	5.250	2.010	1.855
0.9	5.336	5.246	2.013	1.848
1.0	5.337	5.241	2.015	1.841
1.1	5.338	5.238	2.017	1.834
1.3	5.340	5.231	2.021	1.823
1.5	5.342	5.225	2.025	1.813
1.7	5.344	5.220	2.028	1.804
1.9	5.345	5.216	2.029	1.797
2.0	5.346	5.214	2.031	1.794
PDG[19]	5.325	5.279	2.010	1.869
[27]	5.324	5.285	2.009	1.981
[28]	5.324	5.279	2.005	1.965
[29]	5.325	5.277	2.006	1.975

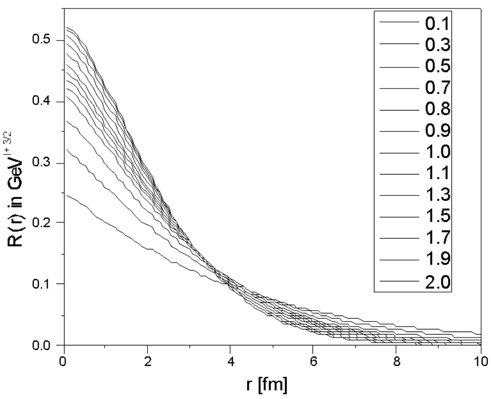

Fig 3: Behaviour of radial wave function for different choices of potential exponent ν for B meson

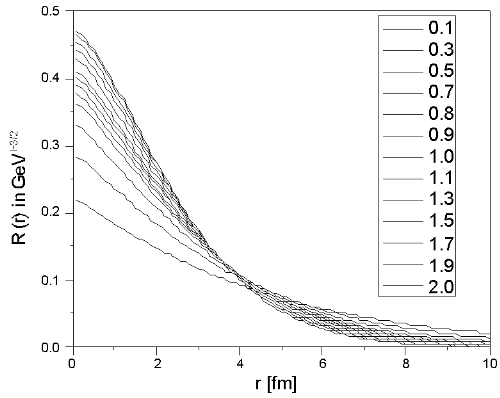

Fig 4: Behaviour of radial wave function for different choices of potential exponent ν for for D meson

RESULTS AND DISCUSSIONS

In the present study we have computed the decay widths for the $B \to D(D^*) + l^+ \nu_l$ channel. For the description of the masses and the wave function of B and D mesons we have adopted phenomenological CPPν model [26]. The computed masses of B and D states are listed in Table 1 with different potential exponent ν and our results are compared with the experimental data and other theoretical model predictions. Our predictions of masses are in accordance with the experimental data with the power index of the potential, ν =0.5 for B states and ν =0.7 for D states. The corresponding masses and the wave functions are employed to compute the Isgur-Wise function. Fig. 5 shows the trend line for the Isgur-Wise function with ω for different choices of the potential index ν. It is clear that the slope of the Isgur-Wise function for ν = 0.1 is larger compared to other choices of ν. The slope of the Isgur-Wise function decreases with increase of the power exponent ν. All the lines correspond to ν ≥ 0.5 tend to merge as ω ≈ 1.0 seen here is an indication of the universality of ξ (ω) as ω = 1.

Our computed branching ratios for semi-leptonic decays of bottom decaying to open charm mesons for different choices of potential exponent ν are listed in Table 2. We found good agreement with experimental values and other theoretical models for B meson decays in the range of potential exponent 0.7 ≤ í ≤ 0.9. Hence, we conclude here the semi-leptonic decays of open heavy meson system occur at same inter-quark potential to that responsible to form bound state.

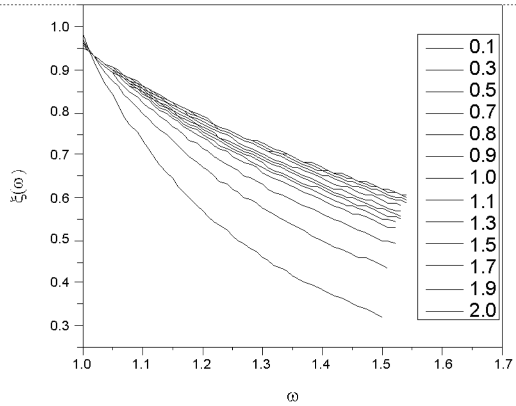

Fig 5: Isgur Wise function ,$\xi(\omega)$ against $\omega = v.v'$

Table 2: Branching ratio (in %) for exclusive semi leptonic decays of open bottom mesons into open charm mesons

Potential Exponent ν	$B \to D + l^+ \nu_l$	$B \to D^* + l^+ \nu_l$
0.1	0.8	3.15
0.3	1.42	4.71
0.5	1.94	5.67
0.7	2.32	6.24
0.8	2.47	6.46
0.9	2.63	6.66
1.0	2.77	6.82
1.1	2.86	6.94
1.3	3.08	7.17
1.5	3.22	7.35
1.7	3.37	7.47
1.9	3.47	7.59
2.0	3.53	7.64
PDG[19]	2.390.12	6.71.3
[30]	2.340.03	5.400.02

At the end, we conclude here that the present study of the properties of heavy light systems based on the non-relativistic Coulomb plus power potential with varying power exponent using numerical approach to solve the Schrödinger equation has provided us a mean to understand the nature of the inter-quark interactions and their parameters that provide us the spectroscopic properties as well as the decay properties of these mesons with the potential index between 0.5 and 0.7. The present study also provides us the importance of the quark mass parameters and the state dependence on the potential strength for the study of the spectral properties.

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