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DECAY WIDTHS OF $B_c \rightarrow J/\Psi\ \pi^+$ IN CPP ν MODEL

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ABSTRACT

The non leptonic properties of B_c meson in the fina lĵ /Ψ state have been studied inthe general framework of non relativistic potential models. The Schrodinger equation correspond to B_C ($\bar{b}$ c) and $Q\bar{Q}$ (c, b) is solved numerically using the Coulomb plus power potential with varying index of the potential ranging from 0.1 to 2.0 as the quark- antiquark interaction. We employ the experimental quarkonia masses to fix the quark masses and the corresponding potential parameters. The ground state mass of the Bc meson is predicted with no additional parameters. The resultant masses and their radial wave functions obtained here are used to compute the decay widths for $B_c \rightarrow J/\Psi\ \pi^+$ channel. The predicted mass is found to be in accordance with the experimental data for the choices power of the potential, $\nu \approx 0.5$ for B_C meson and $\nu \approx 0.5$ for J/Ψ meson. The non leptonic decay width is found to occur at relatively weaker interquark potential compared to that responsible to form the bound state.

Keywords: *non leptonic decay, quarkonia, potential models*

INTRODUCTION

B_C meson is of particular interest amongst the heavy flavoured mesons because of its uniqueness of being only open flavoured heavy meson. The spectroscopy and decay properties of B_C meson have been predicted by various theoretical models [1-9]. Study on the decay properties of B_C meson in final state J/Ψ is of particular interest for experimentalists, because J/Ψ can be observed with the high precision in the leptonic mode $J/\Psi \rightarrow 1^+1^-$. Contrary to the semileptonic modes, the nonleptonic decay mode $B_c \rightarrow J/\psi\pi+$ is outstanding in reconstructing the $B_c$vertex by detecting charged particle tracks and the $B_c$mass can be measured.

NON LEPTONIC DECAY WIDTHS

The estimation of the decay widths $B_c \rightarrow J/\psi\pi +$ has been carried out by various theoretical models [4, 5, 10]. However, the potential models are assumed to give more precise prediction about the decay width due to the fact that the transition form-factors are determined by the overlapping integrals of the decaying and the produced mesons. For the present study we follow the non relativistic formalism given by [10] according to which the total decay width for the $B_c \rightarrow J/\psi\pi+$ channel can be given by,

$$\Gamma(B_c \rightarrow J/\psi\pi^+) = G_F^2 |V_{cb}|^2 \frac{128\pi\alpha_s^2}{81} f_\pi^2 f_{B_c}^2 f_{J/\psi}^2 \left(\frac{M_{B_c}+M_{J/\psi}}{M_{B_c}-M_{J/\psi}}\right)^3 \frac{M_{B_c}^3}{(M_{B_c}-M_{J/\psi})^2 M_{J/\psi}^2} \quad (1)$$

In Eqn. 1,f_{B_C} and $f_{j/\psi}$ are the decay constants and are obtained by parameterizing the matrix elements of weak current between the corresponding mesons and the vacuum as,

$$<O|\bar{Q}\ J^\mu Q|\ P_\mu\ (k)> = if_p k^\mu \quad (2)$$

$$<O|\bar{Q}\ \ J^\mu Q|P_\mu(k)>=f_{pv}M_v\varepsilon^\mu \quad (3)$$

The decay constants appeared in Eqn. 1 can be computed in the nonrelativistic formalism by incorporating first order QCD correction to the Van Royen-Weiskopff formula as,

$$f_{P/V} = \sqrt{\frac{3}{\pi M_{nP/V}}}\ |R_{nP/V}(0)| \left(1 + \frac{\alpha_s}{\pi}\left[\frac{m_q - m_{\bar{q}}}{m_q + m_{\bar{q}}}\right] \ln\frac{m_Q}{m_{\bar{Q}}} - \delta^{P/V}\right) \quad (4)$$

Here, $\delta^p = 8/3$ and $\delta^v = 2$. Hence, by knowing about the spectroscopic parameter and behavior of the wave function at origin we can calculate the decay width for $B_c \rightarrow J/\psi_\Pi +$.

THE PHENOMENOLOGY AND EXTRACTION OF THE SPECTROSCOPIC PARAMETERS

For the description of the quark-antiquark bound states, we adopt the phenomenological Coulomb plus power potential (C P P_ν) expressed as [11, 12]

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar^\upsilon \quad (5)$$

Here, A is the confinement strength of the potential and α_s is the running strong coupling constant which is computed as,

$$\alpha_s(\mu^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right)\ln\frac{\mu^2}{\Lambda^2}} \quad (6)$$

W nber of flavors, μ is the renormalization scale related to the constituent quark mass and Λ is the QCD scale which is taken as 0.150 GeV by fixing $\alpha_s = 0.118$ at the Z−boson mass (91 GeV) [13].

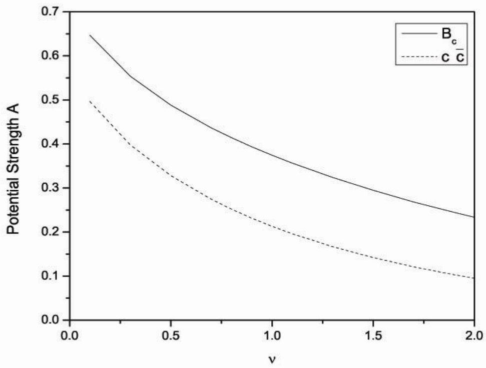

Fig. 1 Potential Strength A (in GeV$^{\nu+1}$) obtained from ground state spin average mass against the choices of potential index, ν (0.1 ≤ ν ≤ 2.0)

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The potential parameter, A of Eqn.4 is similar to the string strength σ of the Cornell potential. We particularly chose to vary ν in our study as very different interquark potentials can provide fairly good description of the mass spectra, while the transitions and other decay properties are very sensitive to the wave functions. And the wave functions vary differently with different choices of interquark potential. Thus in the present study, we vary the potential index ν in the range $0.1 \le \nu \le 2.0$. It can also provide significant understanding of the quark-antiquark interaction in the mesonic states while they undergo a transition or decay through annihilation channels. The different choices of ν here then correspond to different potential forms. So, the potential parameter A expressed in GeV $^{\nu+1}$ can be different for each choices of ν. The model potential parameter A and the mass parameter of the quark/antiquark $(m_1,m_2)$are fixed using the known ground state center of weight (spin average) mass and the hyperfine splitting$(M\,3s_1 - M\,1s_0)$ of the ground state $c\bar{c}$ and $b\bar{b}$ systems respectively. The spin average mass for the ground state is computed for the different choices of ν in the range $0.1 \le \nu \le 2.0$. The spin average or the center of weight mass, M_{CW} is calculated from the known experimental/theoretical values of the pseudoscalar (J =0) and vector (J = 1) mesonic mass as

$$M_{n,cw} = \frac{\Sigma_J (2J+1)\, M_{nJ}}{\Sigma_J (2J+1)} \tag{7}$$

The Schrodinger equation is numerically solved using the mathematica notebook of the Runge-Kutta method [14]. For computing the mass difference between different spin degenerate mesonic states, we consider the spin dependent part of the usual one gluon exchange potential (OGEP) given by [15-19]. Accordingly, the spin dependent part, $V_{SD}(r)$ for the angular quantum number *l* =0 contains only the spin-spin hyperfine interaction given by

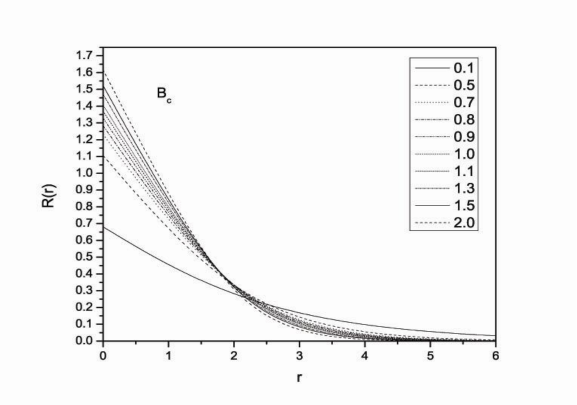

Fig. 2 Behaviour of wave function for different choices of potential index ν for B_c meson.

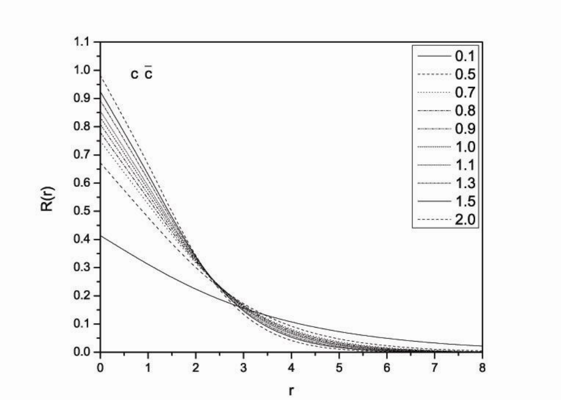

Fig. 3 Behaviour of wave function for different choices of potential index ν for $c\bar{c}$ meson

$$V_{SD}(r) = V_{SS}(r)\left[S(S+1) - \frac{3}{2}\right] \tag{8}$$

The coefficient of this spin-dependent term of Eqn.7 is given by the usual one gluon exchange (OGE) interaction as [17]

$$V_{SS}(r) = \frac{16\pi\alpha_S}{9\,m_1 m_2}\,\delta^{(3)}(\vec{r}) \tag{9}$$

The computed masses of the B_c and $c\bar{c}$ states are listed in Table 1. The spectroscopic parameters thus correspond to the fitted quark masses, the potential strength A, the potential index, ν and the corresponding radial wave functions. The fitted mass parameters are $m_c = 1.28\ \text{GeV}/c^2$, $m_b = 4.4\ \text{GeV}/c^2$ while the potential strength A for each choices of ν are shown in Fig. 1. The numerical solution for the radial wavefunctions thus obtained for the different choices of the potential index, ν are plotted in Fig. 2 and in Fig. 3 in the case of Bc and $c\bar{c}$ systems respectively.

Table-1 Masses (in GeV) of B_c and J/ψ mesons and decay widths of $B_c \to J/\psi_\Pi +$

υ	M_{Bc} GeV	$M_{j/\Psi}$ GeV	f_{Bc} GeV	$F_{j/\Psi}$ GeV	$Bc \to J/\psi_\Pi^+ \times 10^{-15}$ GeV
0.1	6.305	3.076	0.264	0.231	0.21
0.5	6.279	3.088	0.428	0.376	1.48
0.7	6.268	3.093	0.477	0.420	2.31
0.8	6.264	3.095	0.497	0.437	2.74
0.9	6.26	3.097	0.515	0.453	3.16
1.0	6.256	3.099	0.530	0.467	3.57
1.1	6.253	3.100	0.544	0.480	3.97
1.3	6.246	3.103	0.568	0.501	4.75
1.5	6.241	3.106	0.588	0.519	5.47
2.0	6.321	3.111	0.624	0.552	7.04
Expt[20]	6.277	3.097			
[21]	6.27	3.096			
[4]	6.356				1.22
[5]	6.302				1.06
[10]					2.0

RESULTS AND DISCUSSIONS

The computed mass spectra and decay constants for B_c and J/Ψ mesons and the decay widths for $B_c \to J/\psi_\Pi^+$ channel with different choices of potential index υ are shown in Table 1. Our computed mass parameter agrees with the experimental data [20] for the $\upsilon \approx 0.5$ for B_c meson and $\upsilon \approx 0.9$ for J/ Ψ mesons. As experimental results are still awaited for the decay channel $B_c \to J/\psi\pi^+$. We find our results in the range of potential index $0.1 \le \nu \le 0.8$ are in accordance with other theoretical model predictions. It is being observed that the nonleptonic weak decays of B_c meson occur at relatively weaker interquark potential than that corresponds to form a bound state. Such a behavior as seen from the present study is in accordance with our understanding that weak interaction is smaller than strong interaction range.

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