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Editorial Note: *This paper is a corrected version of the paper published earlier as PRAJNA - Journal of Pure and Applied Sciences, Vol. 18: 115 -116 (2010). In the earlier version inadvertently the Statement of Theorem 2.3 was left out and Illustration 2.2 was misplaced. We regret for this and for a complete academic record, we publish this corrected version.*

MEAN LABELING FOR SOME NEW FAMILIES OF GRAPHS

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ABSTRACT

Some new families of mean graphs are investigated. We prove that the step ladder graph, total graph of path P_n are mean graphs. In addition to this we derive that two copies of cycle C_n sharing a common edge admits mean labeling.

Key words: *Mean labeling, Mean graph, Step ladder graph, Total graph*
AMS Subject classification number (2010): 05C78

INTRODUCTION

We begin with simple, finite, connected and undirected graph $G = (V(G),E(G))$ with p vertices and q edges. For all other standard terminology and notation we follow Harary [1]. We will provide brief summary of definitions and other information which serve as prerequisites for the present investigations.

Definition 1.1 Let P_n be a path on n vertices denoted by $(1,1),(1,2),...,(1,n)$ and with $n-1$ edges denoted by $e_1,e_2,...,e_{n-1}$ where e_i is the edge joining the vertices $(1,i)$ and $(1,i+1)$. On each edge e_i,

$i = 1, 2, \ldots, n-1$ we erect a ladder with $n-(i-1)$ steps including the edge e_i. The graph obtained is called a *step ladder graph* and is denoted by $S(T_n)$, where n denotes the number of vertices in the base.

Definition 1.2 The vertices and edges of a graph are called its elements. Two elements of a graph are neighbours if they are either incident or adjacent. The *total graph* of a graph G is denoted by $T(G)$ is a graph with vertex set $V(G) \cup E(G)$ and two vertices are adjacent in $T(G)$ whenever they are neighbors in G.

Definition 1.3 If the vertices are assigned values subject to certain conditions then it is known as *graph labeling.* Graph labeling is one of the fascinating areas of graph theory with wide ranging applications. An enormous body of literature has grown around in graph labeling in last five decades. A systematic study of various applications of graph labeling is carried out in Bloom and Golomb [3]. According to Beineke and Hegde [2] graph labeling serves as a frontier between number theory and structure of graphs. For detailed survey on graph labeling we refer to A Dynamic Survey of Graph Labeling by Gallian [4].

Definition 1.4 A function f is called a *mean labeling* of graph G if $f: V(G) \rightarrow \{0,1,2,....,q\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1,2,....,q\}$ defined as

$$f^*(e = uv) = f(u) + f(v)/2 \qquad \text{if } f(u) + f(v) \text{ is even}$$
$$= f(u) + f(v) + 1/2 \qquad \text{if } f(u) + f(v) \text{ is odd}$$

is bijective. The graph which admits mean labeling is called a *mean graph.*

The mean labeling is introduced by Somasundaram and Ponraj [5] and they proved the graphs P_n, C_n, $P_n \times P_m$, $P_m \times C_n$ etc. admit mean labeling. The same authors in [6] have discussed the mean labeling of subdivision of $K_{1,n}$ for $n \leq 3$ while in [7] they proved that the wheel W_n does not admit the mean labeling for $n \leq 4$. Mean labeling in the context of some graph operations is discussed by Vaidya and Lekha [8]. In the present work three new results corresponding to mean labeling and some new families of mean graphs are investigated.

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MAIN RESULTS

Theorem-2.1: The step ladder graph $S(T_n)$ is a mean graph.

Proof: Let P_n be a path on n vertices denoted by $(1,1),(1,2),...,(1,n)$ and with $n-1$ edges denoted by $e_1,e_2,...,e_{n-1}$ where e_i is the edge joining the vertices $(1,i)$ and $(1,i+1)$. The step ladder graph $S(T_n)$ has vertices denoted by $(1,1),(1,2),...,(1,n)$, $(2,1),(2,2),...,(2,n)$, $(3,1),(3,2),...,(3,n-1),...,(n,1),(n,2)$. In the ordered pair (i,j), i denotes the row (counted from bottom to top) and j denotes the column (from left to right) in which the vertex occurs.

Define $f: V(S(T_n)) \rightarrow \{0,1,2,....,q\}$ as follows.

$$f(i,1) = (n^2 + n - 2) - (i - 1); \qquad 1 \leq i \leq n$$

$$f(1, j) = (n^2 + n - 2) - \sum_{k=1}^{j-1} (n - k) - \sum_{k=2}^{j} [(n + k) - (j - 1)]; \qquad 2 \leq j \leq n$$

$$f(i, j) = (n^2 + n - 2) - \sum_{k=1}^{j-1} (n - k) - \sum_{k=2}^{j} [(n + k) - (j - 1)] - (i - 1); \qquad 2 \leq i, j \leq n \;\&\; j \neq n+2-i$$

$$f(i, n + 2 - i) = i^2 - 2; \qquad 2 \leq i \leq n$$

In view of the above defined labeling pattern f is a mean labeling for the step ladder graph $S(T_n)$. That is, $S(T_n)$ is a mean graph.

Illustration 2.2: The *Figure 1* shows the labeling pattern for $S(T_6)$.

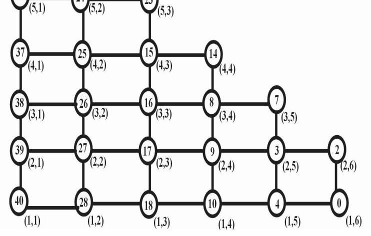

Fig. 1

Theorem-2.3: The total graph of path P_n that is, $T(P_n)$ is a mean graph.

Proof: Let $v_1, v_2, ..., v_n$ be the vertices of path P_n with n-1 edges denoted by $e_1, e_2, ..., e_n$. According to the definition of total graph and two vertices are adjacent in $T(P_n)$ if they are neighbors in P_n. Define $f: V(T(P_n)) \rightarrow \{0,1,2,....,q\}$ as fellows.

$f(v_1) = 0$

$f(v_i) = 4(i-2)+2$; for $2 \leq i \leq n$

$f(e_j) = 4j$; for $1 \leq j \leq n-2$

$f(e_j) = 4j-1$; for $j = n-1$

Thus f provides a mean labeling for $T(P_n)$. That is, $T(P_n)$ is a mean graph.

Illustration 2.4: The labeling pattern of $T(P_5)$ is given in Fig. 2.

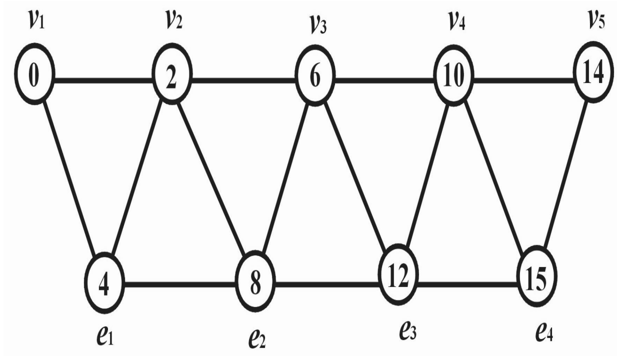

Fig. 2

Theorem-2.5: Two copies of cycle C_n sharing a common edge admit mean labeling.

Proof: Let $v_1, v_2, ..., v_n$ be the vertices of cycle C_n. Consider two copies of cycle C_n. Let G be the graph for two copies of cycle sharing a common edge in which $v_1, v_2, ..., v_n$ is a spanning path. Then V(G) = 2n-2 and E(G) = 2n- 1.

To define $f: V(G) \rightarrow \{0,1,2,....,q\}$ the following two cases are to be considered.

Case 1: n is odd.

Without loss of generality assumes that $e = v_{\frac{n+1}{2}} v_{\frac{3n-1}{2}}$ be the common edge between two copies of C_n.

$f(v_i) = 2(i-1)$; for $1 \leq i \leq n+1/2$

$f(v_i) = 2i-1$; for $n+3/2 \leq i \leq n$

$f(v_i) = 2(2n-2-i)+4$; for $n+1 \leq i \leq 3n-3/2$

$f(v_i) = 2(2n-2-i)+3$; for $3n-1/2 \leq i \leq 2n-2$

Case 2: n is even.

Without loss of generality assume that $e = v_{\frac{n+2}{2}} v_{\frac{3n}{2}}$ be the common edge between two copies of C_n.

$f(v_i) = 2(i-1)$; for $1 \leq i \leq n+2/2$

$f(v_i) = 2i-1$; for $n+4/2 \leq i \leq n$

$f(v_i) = 2(2n-2-i)+4$; for $n+1 \leq i \leq 3n-2/2$

$f(v_i) = 2(2n-2-i)+3$; for $3n/2 \leq i \leq 2n-2$

Then the above defined function f provides mean labeling for two copies of cycle sharing a common edge.

Illustration 2.6 : The Fig. 3 shows the mean labeling pattern for two copies of C_{10} sharing an edge.

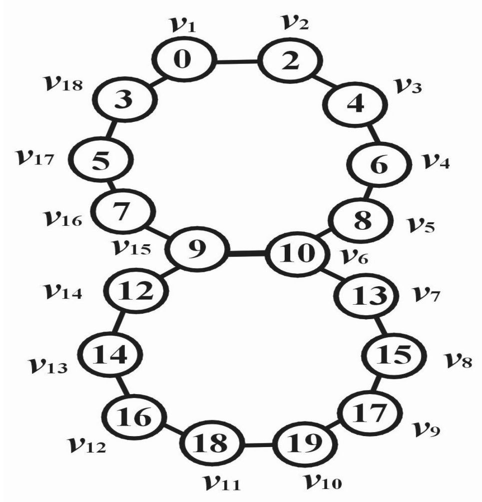

Fig. 3

CONCLUDING REMARKS AND FURTHER SCOPE

As all graphs are not mean graphs it is very interesting to investigate graphs which admit mean labeling. Here we contribute three new families of mean graphs. It is possible to investigate similar results for other graph families and in the context of different labeling techniques.

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