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A MATHEMATICAL MODEL FOR A GROWTH CURVE OF RICE CROP

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ABSTRACT

In this study, a mathematical model is described for a growth curve of rice crop. The parameters of the growth equations are related to the plant. This modeled growth curve is interpreted for growth data from experiments using rice plant of different varieties and the results are reported for conclusion. Here this growth curve also describes the plant dry weight data over a whole season.

***Key words**: Mathematical model, rice crop, growth curve, varieties*

INTRODUCTION

Growth is one of the important properties of living organisms. Changes in a phenotype during the growth period can be modeled via growth curves, such as generalized logistic, logistic or Gompertz growth curves [1]. Behavior of the growth curves can change according to living organisms, the phenotype to be studied and environment to which it is exposed [2]. To evaluate growth data properly, it is required to select a suitable growth curve and its parameters should be interpreted biologically [3]. This study reports on a comparison of logistic growth curve of plant dry weight of three rice varieties that are commonly grown in Tamil Nadu. The aims of this study are: 1) to determine a logistic growth curve for plant dry weight data; and 2) to compare plant dry weight of rice varieties via the growth curve. This study was carried out during 2010. The experiment is conducted at the Tamil Nadu Agricultural University, Coimbatore. Rice varieties namely IR 64, Jaya, Tulasi, which are widely grown in India are used. The field plots used was allocated in a randomized block design (RBD) with five replicate plots per variety. Rice plants dry weights are recorded in every weeks starting from emergence. Measurements are used for five plants given each plot and each variety.

Changes in the plant dry weight of three varieties are modeled through logistic growth curve and are given in mathematical formulation.

MATHEMATICAL FORMULATIONS

The problem is to find a growth curve which will give a good description of plant dry weight data over a whole season. The parameters of the growth equation are physiologically meaningful, and are related either to the environment or to the plant. This growth equation is used to summarize data, and possibly to interpret growth data from experiments using plants of different verities or with different environmental treatments.

ASSUMPTIONS

(I) The plant is completely defined by its dry weight W. The system is described by a single state variable. The variable W is a dependent variable, and varies with time t where t is an independent variable.

(ii) Growth occurs at the expense of a single substrate S.

(iii) The rate of the growth reaction is linearly proportional to the substrate level S, and also to the plant dry weight W, so that the growth is autocatalytic. The rate of the growth reaction is kWS where k is a constant.

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MATHEMATICAL EQUATION

As a consequence of assumption (iii), it follows that

$$\frac{dW}{dt} = kSW \qquad (1)$$

This first –order differential equations cannot be solved, because the substrate level S will vary as growth proceeds. If W and S are measured in the same units, and there is no loss of material when converting S into W by the growth reaction, then

$$dW = -dS \qquad (2)$$

This equation states that an increment in dry weight is exactly matched by a loss in substrate. Equation (2) can be written as d (W+S) = 0, which on integration gives

$$W + S = W_i + S_i = \text{constant}, \qquad (3)$$

Where W_i, and S_i are the values of W and S at time t = 0, and denote the initial conditions. Equations of (2) or (3) simply express the conservation of matter. Since W and S are not allowed to be negative (such values would be physiologically meaningless), it is clear from (3) that W will have its maximum and final value when there is no substrate left, and S =0. Equation (3) may be re written as

$$W + S = W_f = W_i + S_i \qquad (4)$$

Where W_f is the maximum value of W. Substituting for S from Equation (4), Equation (1) becomes

$$\frac{dW}{dt} = k\,(W_f - W)\,W \qquad (5)$$

This equation is a statement of the model in differential form. As the system has only one state variable, only one equation is needed.

Equation (5) is of the variables – separable type, and can be re-arranged in the form

$$\int_{W_i}^{W} \frac{dW}{(W_f - W)W} = \int_0^t k\,dt \qquad (6)$$

The left side is split up into its component partial fractions

$$\int_{W_i}^{W} \frac{1}{W_f}\left(\frac{1}{W_f - W} + \frac{1}{W}\right)dW = \int_0^t k\,dt \qquad (7)$$

and on integration gives

$$\frac{1}{W_f}\left[In\left(\frac{W}{W_i}\right) + In\left(\frac{W_f - W_i}{W_f - W}\right)\right] = kt \qquad (8)$$

This becomes

$$W = \frac{W_i W_f e^{W_f k t}}{W_f - W_i + W_i e^{W_f k t}} \quad (9)$$

For low values of t $W \approx W_i e^{w_f k t}$ *as* $t \to \infty$, $w \to W_f$.

Where

W_i is the initial plant dry weight,

$W_f k$ is the maximum specific growth rate achieved by the plant in early growth when there is no substrate limitation; and

W_f is the final plant weight which is determined [through equation (4)] by the initial amount of substrate available per plant and the initial plant dry weight.

Model fitting and parameter estimation are performed via MATLAB 7 procedure [4] of computing solution to crop growth model.

Results from growth curve fitting of Logistic curve to plant dry weights data and the observed results are summarized in Table 1.The computed plant dry weights and observed values via logistic curve for rice varieties are shown in Fig 1 and Fig 2.

Table 1: Total plant dry weight (Tones/hectare) of rice crop varieties

Total plant dry weight (Tones/hectare)					
IR 64		Jaya		Thulasi	
Observed	**Computed**	**Observed**	**Computed**	**Observed**	**Computed**
3.5	3.5000	3.8	3.8000	2.7	2.7000
5.7	5.7126	6.6	6.6945	4.2	4.2864
7.5	7.5613	8.9	8.9993	5.8	5.8031
8.6	8.6430	10.1	10.1773	6.0	6.8873
9.3	9.3612	10.6	10.6455	7.5	7.5116
9.4	9.4458	10.8	10.8132	7.8	7.8273
9.4	9.4790	10.8	10.8701	7.9	7.9764
9.4	9.4929	10.8	10.8901	8.0	8.0445
9.4	9.4968	10.8	10.8967	8.0	8.0752
9.4	9.4988	10.8	10.8989	8.0	8.0890
9.4	9.4995	10.8	10.8996	8.0	8.0951
9.4	9.4949	10.8	10.8999	8.0	8.0978
9.4	9.4979	10.9	10.9000	8.0	8.0990
9.4	9.5000	10.9	10.9000	8.1	8.0996
9.5	9.5000	10.9	10.9000	8.0	8.0998
9.5	9.5000	10.9	10.9000	8.1	8.0999
9.5	9.5000	10.9	10.9000	8.1	8.1000
9.5	9.5000	10.9	10.9000	8.0	8.1000

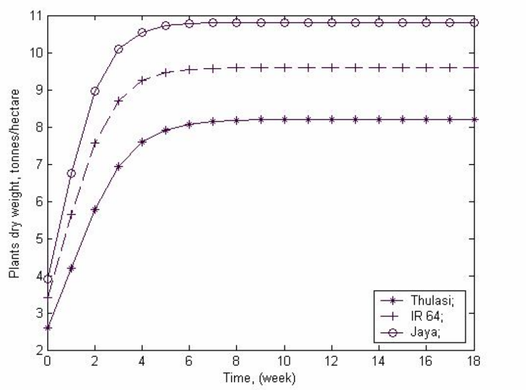

Fig.1: Growth curves of three varieties of rice crop – (Computed)

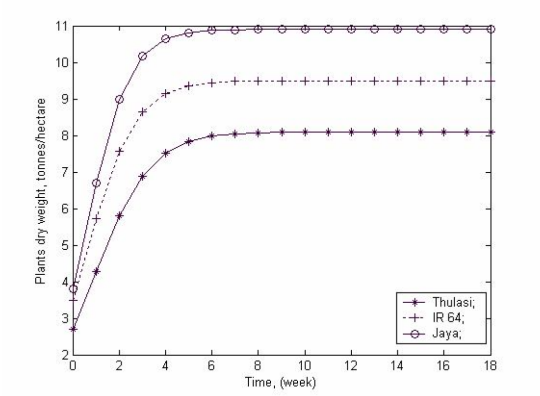

Fig.2: Growth curves of three varieties of rice crop – (Observed)

CONCLUSION

In this study, growth curve is described for plant dry weight of three varieties of rice crop by Mathematical modeling and it is found that the Jaya variety has more dry weight and has good agreement with the observed results.

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REFERENCES

[1] Thornely, J.H.M and France, J. (2007**)** *Mathematical Models in agriculture: Quantitative methods foe plant, animal and ecological sciences***,** London,UK, pp.1-887.

[2] Wang, E. and Smith, C. (2004) Modeling the growth and water uptake function of plant root system: a review. *Aust. J. Agric. Res.* **55** : 501–523.

[3] Christopher Teh (2006) *Introduction to mathematical Modeling of crop growth: How the equation are derived and assembled into a computer Program.* Brown Walker press, Boca Raton, pp. 1-280.

[4] Chapra, S. C (2007) *Applied Numerical Methods with MATLAB for Engineers and Scientist* , Tata McGrraw-Hill publishing company limited, New Delhi, pp. 1-588.