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SARDAR PATEL UNIVERSITY
M.Sc. (Statistics) 3rd Semester Examination
2018
Wednesday, 24th October
2:00 p.m. to 5:00 p.m.
Course No. PS03CSTA02/22
(Multivariate Analysis)

Note: Figures to the right indicate full marks of the questions. **(Total Marks: 70)**

1 Write the correct answer (each question carries one mark). **08**

(a) A random vector $\underline{X}$ is said to have multivariate normal distribution iff

(A) its pdf is of the form $(2\pi)^{-p/2}|\Sigma|^{-1/2}\exp\{-\frac{1}{2}(\underline{x}-\underline{\mu})'\Sigma^{-1}(\underline{x}-\underline{\mu})\}$

(B) every linear function of $\underline{X}$ has univariate normal distribution.

(C) marginal distribution of any subvector of normal vector is normal.

(D) conditional distribution, given remaining components, is normal.

(b) Under the standard Wishart distribution, the distribution of sample generalized variance is

(A) product of independent Wishart distributions

(B) product of independent chi-square distributions

(C) product of independent beta distributions

(D) none

(c) Suppose $\underline{X}_1$ and $\underline{X}_2$ are iid r.vs. from $N_p(\underline{0},\Sigma), \Sigma > 0$. Which is true

(A) $\underline{X}_1\underline{X}_1' + \underline{X}_2\underline{X}_2' \sim 2\chi^2_p$

(B) $\underline{X}_1\underline{X}_1' + \underline{X}_2\underline{X}_2' \sim \chi^2_{2p}$

(C) $\underline{X}_1\underline{X}_1' + \underline{X}_2\underline{X}_2' \sim 2W_p(\Sigma, 1)$

(D) $\underline{X}_1\underline{X}_1' + \underline{X}_2\underline{X}_2' \sim W_p(\Sigma, 2)$

(d) Suppose $\underline{X}_1, \dots, \underline{X}_N$ are iid r.vs. from $N_p(\underline{0},\Sigma), \Sigma > 0$. For every $\underline{l} \in \mathcal{R}^p$, $\sum_{i=1}^{N} \underline{l}'\underline{X}_i\underline{X}_i'\underline{l}$ follows

(A) a normal distribution

(B) a chi-square distribution

(C) a Wishart distribution

(D) none

(e) Let $\overline{\underline{X}}$ and S be the sample mean vector and unbiased covariance matrix of a r.s. of size N from $N_p(\underline{\mu},\Sigma), \Sigma > 0, N > p$. Then Hotelling's T^2 statistic may be constructed as

(A) $(N-1)(\overline{\underline{X}}-\underline{\mu})'S^{-1}(\overline{\underline{X}}-\underline{\mu})$

(B) $N(\overline{\underline{X}}-\underline{\mu})'S^{-1}(\overline{\underline{X}}-\underline{\mu})$

(C) $\frac{1}{N-1}(\overline{\underline{X}}-\underline{\mu})'S^{-1}(\overline{\underline{X}}-\underline{\mu})$

(D) $\frac{1}{N}(\overline{\underline{X}}-\underline{\mu})'S^{-1}(\overline{\underline{X}}-\underline{\mu})$

(f) A square of multiple correlation coefficient is distributed as
(A) beta kind I
(B) beta kind II
(C) chi-square
(D) Wishart

(g) Suppose $\underline{X}_1^{(i)},\ldots,\underline{X}_{N_i}^{(i)}$ are independent r.vs. from $N_p(\underline{\mu}_i,\Sigma),\Sigma>0$. Let $\overline{\underline{X}}$ be the combined mean vector and $B=\sum_{i=1}^{k}N_i(\underline{X}_i-\overline{\underline{X}})(\underline{X}_i-\overline{\underline{X}})'$. The distribution of B is (A) $W_p(\Sigma,\mathrm{k}-1)$
(B) $W_p(\Sigma,\mathrm{k}-1)$ provided $\underline{\mu}_1=\cdots=\underline{\mu}_k$
(C) $W_p(\Sigma,\mathrm{n})$ where $n=\sum_{i=1}^{k}(N_i-1)$
(D) none of the above

(h) A Wilk's Λ statistic is the ratio of two independent
(A) Wishart matrices
(B) Wishart matrices divide by respective d.f.
(C) generalized variances
(D) none of the above.

2 Answer any SEVEN of the following (each question carries two marks) **14**

(a) Let $\underline{X}\sim N_p(\underline{\mu},\Sigma)$. Obtain its density when Σ is n.n.d.

(b) Let $A\sim W_p(\Sigma,m)$ and $B\sim W_p(\Sigma,n)$ be independent. Obtain the distribution of $A+B$.

(c) Let $V\sim W_p(\Sigma,n)$. Obtain the distribution of (i) V^{-1} and (ii) $V_{11\cdot 2}^{-1}$.

(d) In usual notations, show that $V>0$ if and only if $N>p$.

(e) Obtain the null distribution of a sample multiple correlation coefficient.

(f) Using one of the properties of Wishart distribution, obtain the distribution of the Hotelling's T^2 variable.

(g) State Box theorem.

(h) Let $V_i\sim W_p(\Sigma,n_i), i=1,\ldots,k$ be independently distributed. Let $V=V^{1/2}V^{1/2}$ and $V_i=V^{1/2}L_iV^{1/2}$, $i=1,\ldots,k-1$. Obtain the joint distribution of $(L_1,\ldots,L_{k-1})$.

(i) Define multivariate regression model and obtain the MLEs of the parameters of this model.

3 (a) Define multivariate normal distribution. Show that **06**
(i) the distribution of $\underline{X}$ can be determined from the distribution of univariate normal.

(ii) $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ if and only if $\underline{X}$ can be written as $\underline{X} = \underline{\mu} + B\underline{Y}$, $BB' = \Sigma$, where B is p x k matrix of rank k and $\underline{Y} \sim N_k(\underline{0}, I)$.

(b) Obtain the conditional distribution of subvector of normal vector given the remaining components. **06**

OR

(b) Define symmetric normal distribution and obtain the distribution of intra-class correlation.

4 (a) State and prove Bartlett's decomposition lemma. Hence obtain the density of standard Wishart distribution. **06**

(b) Define inverted Wishart distribution and prove its import theorem. From this theorem deduce that **06**

(i) $\sigma^{pp}/v^{pp} \sim \chi^2_{n-p+1}$ where σ^{pp} is the $(p,p)^{th}$ element of Σ^{-1} (V^{-1}) and

(ii) $\underline{h}'\Sigma^{-1}\underline{h}/\underline{h}'V^{-1}\underline{h} \sim \chi^2_{n-p+1}$, where $\underline{h}$:px1 is a real vector.

OR

(b) State and prove residual theorem.

5 (a) Discuss test of contrasts. Using this derive the Hotelling's T^2 test for Randomized Block Design (RBD) when the underlying assumptions are not met. **06**

(b) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma), \Sigma > 0$. Construct the test for testing of hypothesis $H: \underline{\mu} = \underline{\mu}_0$,where $\underline{\mu}_0$ is specified and Σ is unknown, either using LRT or Roy's union-intersection principle. **06**

OR

(b) Discuss Fisher-Behran problem in multivatiate analysis.

6 (a) Develop the sprericity test. **06**

(b) Let $V \sim W_p(\Sigma, n), \Sigma > 0, n \geq p$ and let $V = (V_{ij}), \Sigma = (\Sigma_{ij})$ where $V_{ij}: p_i \times p_j, \Sigma_{ij}: p_i \times p_j$, $i, j = 1, \dots, k$ and $p_1 + \cdots + p_k = p$. Obtain the asymptotic distribution of LRT for testing $H: \Sigma = \begin{pmatrix} \Sigma_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Sigma_{kk} \end{pmatrix}$ against $K \neq A$. **06**

OR

(b) For multivariate regression model develop LRT for testing the general hypothesis.

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