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SARDAR PATEL UNIVERSITY

M.Sc. 3rd Semester Examination

2019

Friday, 22nd March

2:00 p.m. to 5:00 p.m.

STATISTICS COURSE No. PS03CSTA22

(Multivariate Analysis)

Note: Figures to the right indicate full marks of the questions. **(Total Marks: 70)**

1 Wright the correct answer (each question carries one mark). **08**

(a) In multivariate analysis we assume that the under lying distribution is multivariate normal. The reason(s) is(are)

(A) it has very nice properties (B) mathematics is tractable

(C) derivations run parallel to the univariate theory. (D) all of the above

(b) In symmetric normal distribution all the variables

(A) have first moments identical (B) have first moments identical

(C) are independent variables (D) (A) and (B) but not (C)

(c) A nonsingular Wishart distribution has density if

(A) No. of observations exceed no. of parameters (B) Covariance matrix is p.d.

(C) Covariance matrix is p.s.d. (D) none of the above

(d) Let $\underline{X} \sim N(\underline{\mu}, \Sigma)$, $\Sigma > 0$. Then the distribution of $(\underline{X} - \underline{\mu})(\underline{X} - \underline{\mu})'$ is

(A) $\chi_p^2(\underline{\mu}'\Sigma^{-1}\underline{\mu})$ (B) χ_p^2 (C) $W_p(n, \Sigma)$ (D) $W_p(1, \Sigma)$

(e) Hotelling's T^2 statistic can be used for

(A) CRD (B) RBD (C) LSD (D) Factorial designs

(f) A sample multiple correlation coefficient is distributed as

(A) beta kind I (B) beta kind II (C) chi-square (D) Wishart

(g) Which test is called MANOVA test in case of k independent MNDs.

(A) $\underline{\mu}_1 = \cdots = \underline{\mu}_k$ when $\Sigma_1 = \cdots = \Sigma_k$ (B) $\Sigma_1 = \cdots = \Sigma_k$

(C) $\underline{\mu}_1 = \cdots = \underline{\mu}_k$ and $\Sigma_1 = \cdots = \Sigma_k$ (D) none

(h) A Wilk's Λ statistic is the ratio of two independent

(A) Wishart matrices (B) Wishart matrices divide by respective d.f.

(C) generalized variances (D) none of the above.

2 Answer any SEVEN of the following (each question carries two marks) **14**

(a) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$. Show that every linear function of $\underline{X}$ is normally distributed. Is the converse true? Justify your answer.

(b) In usual notations, show that $V > 0$ if and only if $N > p$.

(c) Let $A \sim W_p(\Sigma, m)$ and $B \sim W_p(\Sigma, n)$ be independent. Obtain the distribution of $A + B$.

(d) Let $V \sim W_p(\Sigma, n)$. Obtain the distribution of $V_{11\cdot 2} = V_{11} - V_{12}V_{22}^{-1}V_{21}$.

(P.T.O.)

(e) Define inverted Wishart distribution.

Let $V \sim W_p(\Sigma, n)$ and $U = V^{-1}$. Show for A:k x p of rank k that

$$(AUA')^{-1} \sim W_k((A\Sigma^{-1}A')^{-1}, n-p+k)$$

(f) Define sample multiple correlation coefficient and derive its null distribution.

(g) Using one of the properties of Wishart distribution, obtain the distribution of the Hotelling's T^2 variable.

(h) Specifying the conditions, in usual notations, show that $B = \sum_{i=1}^{k} N_i(\underline{X}_i - \overline{\underline{X}})(\underline{X}_i - \overline{\underline{X}})'$ is distributed as $W_p(\Sigma, k-1)$.

(i) Define multivariate regression model and obtain the MLEs of the parameters of the model.

3 (a) Show that variance–covariance matrix Σ is non-negative definite (n.n.d.). Obtain the density of multivariate normal distribution when Σ is n.n.d. **06**

(b) Show that $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ if and only if $\underline{X}$ can be written as $\underline{X} = \underline{\mu} + B\underline{Y}$, $BB' = \Sigma$, where B is p x k matrix of rank k and $\underline{Y} \sim N_k(\underline{0}, I)$. **06**

OR

(b) Define symmetric normal distribution and obtain the distribution of intra-class correlation.

4 (a) Define standard Wishart matrix. Derive its density. **06**

(b) State and prove residual theorem. **06**

OR

(b) If $X: p \times N$ is a data matrix from $N_p(\underline{0}, \Sigma)$, and if $C: N \times N$ is a symmetric real matrix then show that $XCX' \sim W_p(\Sigma, r)$ iff $C = C^2$, where $r = tr(C) = rank(C)$.

5 (a) Define Hotelling's T^2 statistic and derive its density using transformation of variable technique. **06**

(b) Discuss Fisher-Behran problem in multivatiate analysis. **06**

OR

(b) Discuss application of Hoteling's T^2 statistic in Randomized Block Design (RBD).

6 (a) Construct the LRT for test of independence of subvectors of normal vector and derive its asymptotic distribution. **06**

(b) Describe the method of estimation of the parameters in multivariate regression model when regression matrix is not a full rank matrix. **06**

OR

(b) Write a note on one-way MANOVA.

—X—