[101/102] SEAT No.________

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SARDAR PATEL UNIVERSITY
M.Sc. (Statistics) 2nd Semester Examination
2018
Saturday, 27th October
10:00 a.m. to 01:00 p.m.
Course No. PS02CSTA03/ PS02CSTA23
(Statistical Inference II)

Note: Figures to the right indicate marks. (Total marks: 70)

1 Write the correct answer (each question carries one mark). **08**

(a) Type II error is defined as
(A) the probability of accepting the null hypothesis H_0 when it is false
(B) Rejecting H_0 when it is true
(C) Accepting H_0 when it is false
(D) the probability of rejecting H_0 when it is true

(b) An MP test for testing simple null versus simple alternative for a distribution involving a single parameter is also UMP test for testing simple null versus composite alternative if
(A) distribution has MLR property (B) test is unbiased
(C) CR is free from alternative values of the parameter (D) none

(c) A UMP test becomes UMP unbiased test if
(A) power function is continuous (B) θ belongs to boundary set
(C) a test is level α test (D) none

(d) A UMP test exists for simple null vs two-sided alternative if
(A) $\partial L(x;\theta)/\partial\theta = \text{constant}$ (B) distribution is one-parameter EFD
(C) critical region is free from alternatives (D) power function is continuous

(e) If power function of every test is continuous then an α level UMP similar test is
(A) UMP (B) UMPU
(C) having Neyman structure (D) none.

(f) LRT cannot be applied to test a hypothesis for
(A) goodness-of-fit (B) dependent samples
(C) multi-parameters (D) multivariables

(g) To determine the cut-off points A and B in SPRT we require to know
(A) null distribution of a test statistic
(B) distribution of a test statistic under alternative hypothesis
(C) error probabilities
(D) none

(h) The kernel for estimating squared mean is
(A) X_i (B) X_iX_j
(C) $X_i^2 - X_iX_j$ (D) X_i^2.

2 Answer any SEVEN of the following (each question carries two marks) **14**

(a) State the Neyman-Pearson lemma.

(b) Define Monotone Likelihood Ratio (MLR). Give at least one example of a distribution which does and which does not have the MLR property.

(PTO)

(c) Let $X_1,\dots,X_n$ be a random sample of size n from

$$f(x,\theta)=\frac{1}{\theta}\exp\{-\frac{x}{\theta}\},\quad x>0,\theta>0$$

Obtain a UMP test of size α for testing $H_0:\theta\le\theta_0$ against $H_1:\theta\ge\theta_0$.

(d) Suppose you have a sample of size n from a distribution having MLR property in $T(\underline{x})$. Write down the UMP test for testing $H_0:\mu\le\mu_0$ against $H_1:\mu>\mu_0$.

(e) Define unbiased and UMP unbiased test.

(f) Prove that α-similar test is unbiased test.

(g) Define SPRT. How does it differ from Neyman-Pearson test?

(h) For the SPRT with stopping bounds (A, B) and strength (α,β) show that

$$A\le\frac{1-\beta}{\alpha}\ \text{ and }\ B\ge\frac{\beta}{1-\alpha}$$

(i) Define U-Statistic and state its properties.

3 (a) Define randomized test. By way of an example show why it is needed? **06**

(b) Define UMP test. Derive an UMP test for testing composite null hypothesis against composite alternative for distributions possessing MLR property. **06**

OR

(b) Show that based on a sample of size n from $N(\theta, 1)$, UMP test does not exist for testing $H_0:\theta=\theta_0$ against $H_1:\theta\ne\theta_0$.

4 (a) Define α-similar test and Neyman-structure test. Show that every Neyman-structure test is α-similar test. Why do we need these concepts? **06**

(b) Let $X_1,\dots,X_n$ be a r.s. from $U(\theta_1,\theta_2)$ distribution. Obtain a test for $H:\theta_1\le 0$ vs $K:\theta_1<0$ and name the derived test. **06**

OR

(b) Consider a random sample $X_1,\dots,X_n$ from $N(\theta,\sigma^2)$, σ^2 unknown. Using ancillary statistic find out a UMP unbiased size-α test for testing $H:\mu\le\mu_0$ against $K:\mu>\mu_0$.

5 (a) Define UMA and UMA unbiased confidence bounds and obtain one of them for the parameter θ of $f(x;\theta),\theta\in\Omega$. **06**

(b) Define Likelihood Ratio Test (LRT) and specifying the under lying assumptions obtain its asymptotic distribution in case of simple hypotheses. **06**

OR

(b) Let $X_1,\dots,X_n$ be a random sample of size n from $N(\mu,\sigma^2)$. Obtain the LRT test for testing $H_0:\mu=\mu_0$ against $H_1:\mu\ne\mu_0$.

6 (a) State and prove Ward's fundamental identity. **06**

(b) Define the following terms and give illustration for each term. **06**
Estimable parameter, Degree of a parameter, Kernel of a parameter,
Symmetric kernel, U-statistic

OR

(b) Obtain exact and asymptotic expressions of the variance of one-sample U-statistic.

—X—