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SARDAR PATEL UNIVERSITY
M.Sc.(II Semester) Examination
2012
Wednesday, 5th December
10:30 a.m. to 1:30 p.m.
Course No. PS02CSTA03
(Statistical Inference II)

Note: Figures to the right indicate marks. (Total marks: 70)

1 Write the correct answer (each question carries one mark). 8

(a) A UMP test becomes UMP unbiased test if
(i) power function is continuous (ii) θ belongs to boundary set
(iii) a test is level α test (iv) none

(b) A UMP test exists for simple null vs two-sided alternative if
(i) $\partial L(x;\theta)/\partial\theta$ = constant
(ii) distribution is one-parameter EFD
(iii) critical region is free from alternatives
(iv) power function is continuous

(c) For multiparameter EFD a best test exits for composite alternative and is of the type
(i) MP test (ii) UMP test (iii) UMPU test (iv) none of the above

(d) An MP test, for testing simple null versus simple alternative for a distribution involving a single parameter, is also UMP test for testing simple null versus composite alternative if
(i) distribution has MLR property
(ii) test is unbiased
(iii) critical region is free from alternatives value of the parameter
(iv) none

(e) To determine the cut-off points A and B in SPRT we require to know
(i) null distribution of a test statistic
(ii) distribution of a test statistic under alternative hypothesis
(iii) error probabilities
(iv) none

(f) The ratio of likelihood function under H_0 and under the entire parameter space is called
(i) probability ratio (ii) sequential probability ratio
(iii) likelihood ratio (iv) none of these

(g) A test $\phi(x)$ such that $0 \le \phi(x) \le 1$ is a
(i) non randomized test (ii) randomized test
(iii) most powerful test (iv) none of these

(h) A degree of an estimable parameter is
(i) smallest sample size

(ii) smallest sample size for which the parameter is estimable
(ii) an unbiased estimator based on smallest sample size
(iv) none.

2 **Answer any FIVE of the following (each question carries two marks)** 14

(a) Show that MP test is not unique.

(b) Define Monotone Likelihood Ratio (MLR). Give at least one example of a distribution which does and which does not have the MLR property.

(c) Let $X_1,\ldots,X_n$ be a random sample of size n from

$$f(x,\theta)=\frac{1}{\theta}\exp\{-\frac{x}{\theta}\},\quad x>0,\theta>0$$

Obtain a UMP test of size α for testing $H_0:\theta\le\theta_0$ against $H_1:\theta\ge\theta_0$.

(d) Define: (i) Similar test, (ii) Neyman structure test.
Prove that every Neyman structure test is a similar test.

(e) Define an ancillary statistic. Give an example.

(f) State Basu's theorem. State its applications in testing of hypothesis.

(g) State Wald's theorem.

(h) For the SPRT with stopping bounds (A, B) and strength (α,β) show that

$$A\le\frac{1-\beta}{\alpha}\quad\text{and}\quad B\ge\frac{\beta}{1-\alpha}$$

(i) Give the names of any four tests which are derived as likelihood ratio tests.

3 (a) State Neyman-Pearson (N-P) lemma and prove its sufficiency part only. 6

(b) Define UMP test. Derive an UMP test for testing composite null hypothesis against composite alternative for distributions possessing MLR property. 6

OR

(b) Let $X_1,\ldots,X_n$ be a r.s. from $U(\theta_1,\theta_2)$ distribution. Obtain a test for $H:\theta_1\le 0$ vs $K:\theta_1<0$ and name the derived test. 6

4 (a) Let X has an exponential family of distribution. Show that for testing $H:\theta_1\le\theta\le\theta_2$ vs $K:\theta<\theta_1$ or $\theta>\theta_2$ with size α, a test of the form 6

$$\varphi(x)=\begin{cases}1 & \text{if}\quad T(x)<c_1 \text{ or } T(x)>c_2,\\ \gamma_i & \text{if}\quad T(x)=c_i,\ i=1,2\ (c_1<c_2),\\ 0 & \text{if}\quad c_1<T(x)<c_2,\end{cases}$$

is UMP unbiased, where the constants to be determined such that

$$E_{\theta_1}\varphi(x)=\alpha\quad\text{and}\quad E_{\theta_2}\varphi(x)=\alpha$$

(b) Explain the concepts: Similar test and Neyman structure test. 6

Let $P(x)=c(\theta)\exp\{\theta\, t(x)\}h(x)$ be the p.d.f. of a random variable X. Obtain UMPU test of size α for testing $H_0:\theta=\theta_0$ against $H_1:\theta\neq\theta_0$ using the above concepts.

OR

(b) Let $X_1,\ldots,X_n$ be a random sample of size n from $N(\mu,\sigma^2)$. Obtain the UMPU for testing $H_0:\sigma^2=\sigma_0^2$ against $H_1:\sigma^2\neq\sigma_0^2$ using ancillary statistic. 6

5 (a) Define UMA confidence bounds. Giving suitable example, explain how it is related to a UMP test. 6

(b) Define Likelihood Ratio Test (LRT) and specifying the under lying assumptions obtain its asymptotic distribution in case of simple hypotheses. 6

OR

(b) Let $X_1,\ldots,X_n$ be a random sample of size n from $N(\mu,\sigma^2)$. Obtain the LRT test for testing $H_0:\mu=\mu_0$ against $H_1:\mu\neq\mu_0$.

6 (a) If Z is an r.v. such that 6

$$P\{Z>0\}>0 \text{ and } P\{Z<0\}>0,$$

$$M(t)=E(e^{tZ}) \text{ exist for any real value t,}$$

and $E(Z)\neq 0$, then show that there exist a $t^*\neq 0$ such that $M(t^*)=1$. Moreover, if $E(Z)<0$, then $t^*>0$; and if $E(Z)>0$, then $t^*<0$.

(b) State and prove Ward's fundamental identity. 6

OR

(b) Define U-statistic and derive its asymptotic distribution. 6

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