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SARDAR PATEL UNIVERSITY
M.Sc. (II Semester) Examination
2012
Friday, 30th November
10:30 a.m. to 1:30 p.m.
STATISTICS COURSE No. PS02CSTA01
(Stochastic Processes)

Note: Figures to the right indicate full marks of the questions. **(Total Marks: 70)**

1 **Attempt all, write correct answers** **08**

(i) The transition probability p_{11}^{2} for MC with TPM $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$ is

a) 5/9 b) 1/9
c) 4/9 d) 2/3

(ii) The probability of no events for Poisson process with rate $\lambda = 1/5$ during 2 unit time is

a) exp(-5/2) b) exp(-2/5)
c) exp(-1/5) d) exp(-1/10)

(iii) A branching process has probability distribution {1/4, ¼, ½}. What is the probability that the population will die out if it initially consists of *n* individual?

a) 1/2 b) n/2
c) 2/n d) $(1/2)^n$

(iv) The conditional distribution of k (<n) events must have occurred in time interval of length s (<t) given *n* Poisson process events have occurred in time interval of length *t* is

a) Binomial (n, s/t) b) Uniform [0, t]
c) Geometric (s/t) d) Exponential

(v) Birth-death process with which of the following rates relationship has constant mean population size?

a) birth rate = death rate b) birth rate < death rate
c) birth rate > death rate d) Immigration rate= death rate

(vi) The probability of extinction for a Immigration death process with initially no member and immigration rate 1, death rate 1, is

a) 1 b) Exp(-1)
c) Exp(-2) d) 1/2

(vii) The { X(t)} is a standard Weiner process then ____ is also a standard Weiner process

a) $\{tX(1/t)\}$ b) $\{\sigma X(t/\sigma^2)\}$
c) $\{tX(t)\}$ d) $\{(\sigma/\sqrt{t})X(t/\sigma^2)\}$

(viii) The steady state mean of Ornstein-Uhlenbeck process is

a) 0
b) positive
c) ∞
d) negative

2 **Attempt ANY 7, each carries 2 marks** **14**

(a) Define TPM of Ehrenfest Markov Chain of order 3 and find p_{22}^{2}.

(b) Give an example of a three state absorbing Markov Chain having single absorbing state.

(c) Show that in context of Markov chain recurrence is a class property.

(d) Show that $P_n(s) = P(P_{n-1}(s))$ in context of discrete branching process.

(e) Show that the interval between two successive occurrences of a Poisson process having parameter λ has a negative exponential distribution with mean $1/\lambda$, $\lambda>0$.

(f) Give an example of two state Markov Chain having one step transition probabilities equal to the stationary probabilities.

(g) Prove that, linear birth death process will extinct if death rate is higher than the birth rate.

(h) Narrate the interpretation of the M/M/1 queue having mean equal to one.

(i) State five properties of Weiner process.

(j) Give classification of stochastic processes giving one each real life example.

3(a) Derive maximum likelihood estimator of transition probabilities of finite Markov Chain. State the useful results about test of transition probability matrix. **06**

3(b) Prove that, symmetric random walk Markov Chain is recurrent. **06**

OR

3(b) Consider the Markov chain having states 0, 1, 2, 3, 4 and

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/2 \end{bmatrix}$$

Determine the recurrent states and positive recurrent states.

4(a) Define Poisson cluster Process. Obtain its mean and variance. **06**

4(b) Suppose that people immigrate into a territory at a Poisson rate one per day. What is the expected time until the twelfth immigrant arrives? What is the probability that the elapsed time between tenth and the eleventh arrival exceeds four days? **06**

OR

4(b) Establish that the sum of two independent Poisson processes is a Poisson process but the difference is not a Poisson process.

5(a) Define pure birth process. Hence deduce difference-differential equations and model for Yule-Fury process. **06**

5(b) Describe the Kendall process in reference to linear growth process. **06**

OR

5(b) Show in usual notation and understanding that $P_{10} = \dfrac{\lambda_9 \lambda_8 \ldots \lambda_0}{\mu_{10} \mu_9 \ldots \mu_1} P_0$.

6(a) Define Weiner process giving its transition probability density function. Verify whether Weiner process is covariance stationary or not. **06**

6(b) If $\{X(t)\}$ is a standard Weiner process then establish that$\{tX(1/t)$ is also Weiner process. **06**

OR

6(b) Derive transition probability density function for Ornstein-Uhlenbeck process.