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SARDAR PATEL UNIVERSITY
M. Sc. (II Semester) Examination
2019
Saturday, 23rd March
10:00 a.m. to 1:00 p.m.
Course No. PS02CSTA23/03
(Statistical Inference II)

Note: Figures to the right indicate marks. (Total marks: 70)

1 **Write the appropriate answer (each question carries one mark).** **08**

(a) Neyman –Pearson lemma provides

(A) an unbiased test (B) a most powerful test

(C) an admissible test (D) minimax test

(b) Consider the distribution of X under $H: X \sim P_0$ vs $K: X \sim P_1$ as

x	0	1	2
P_0	0.90	0.08	0.02
P_1	0.50	0.40	0.10

The size the test $\varphi(x) = \begin{cases} 1 & \text{if } x > 1 \\ 1/4 & \text{if } x = 1 \\ 0 & \text{if } x < 1 \end{cases}$ is ________.

(A) 0.60 (B) 0.20 (C) 0.05 (D) 0.04

(c) Consider the distribution of X under $H: X \sim P_0$ vs $K: X \sim P_1$ as

x	0	1	2
P_0	0.10	0.40	0.50
P_1	0.02	0.08	0.90

Let $\varphi_1(x) = \begin{cases} 1 & \text{if } x = 0, 2 \\ 0 & \text{otherwise} \end{cases}$ and $\varphi_2(x) = \begin{cases} 1 & \text{if } x = 2 \\ 1/4 & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$

Then this shows that

(A) class of tests of size $\alpha = 0.6$ is not empty (B) both the test have same size

(C) both the test have same power (D) MP test is not unique

(d) A UMP test exists for simple null vs two-sided alternative if

(A) $\partial L(x;\theta)/\partial\theta =$ constant (B) distribution is one-parameter EFD

(C) critical region is free from alternatives (D) power function is continuous

(e) For testing $H: \theta = \theta_0$ against $K: \theta \neq \theta_0$ in (k + 1)-parameter EFD, the $E_\theta T\varphi(T) = \alpha E_\theta \varphi(T)$ is redundant if

(A) T is complete sufficient (B) distribution of T is symmetric

(C) T is an ancillary statistic (D) none of these

(f) The LRT for testing $H: \theta = \theta_0$ vs $K: \theta = \theta_1$ rejects H whenever

(A) $L(x;\theta_1)/L(x;\theta_0) \leq c$ (B) $L(x;\theta_1)/L(x;\theta_0) \geq c$

(C) $\max L_H(x;\theta)/\max L_\Omega(x;\theta) \geq c$ (D) $\max L_H(x;\theta)/\max L_\Omega(x;\theta) \leq c$

(g) The more OC means

(A) the larger probability of accepting H_0
(B) the larger probability of rejecting H_0
(C) the larger probability of accepting H_1
(D) none

(h) The asymptotic variance of one-sample U-statistic is
(A) $m^2\xi_1$ (B) $m^2\xi_1/n$ (C) ξ_1 (D) none

2 Answer any SEVEN of the following (each question carries two marks) **14**

(a) State Neyman-Pearson (N-P) lemma and prove its uniqueness part.

(b) Testing $H:\theta = \theta_0$ vs $K:\theta = \theta_1 (\theta_1 > \theta_0)$ for $U(0,\theta)$ distribution two tests

$$\varphi_1(t) = \begin{cases} 1 & \text{if } t > \theta_0 \\ \alpha & \text{if } 0 < t < \theta_0 \end{cases} \quad \text{and} \quad \varphi_2(t) = \begin{cases} 1 & \text{if } t > \theta_0(1-\alpha)^{1/n} \\ 0 & \text{otherwise} \end{cases}$$

of same size are given, where $T = \max\{X_i, i = 1, \dots, n\}$. Show that they are same.

(c) For the family of distribution $f(x;\theta) = \theta(\theta + x)^{-2}, x \geq 0, \theta > 0$, obtain the UMP test based on a sample of size one for testing $H:\theta = 2$ against $K:\theta > 2$.

(d) Show that, for the one-parameter exponential family, there exists a UMP test of the hypothesis $H_0: \theta \leq \theta_1$ or $\theta \geq \theta_2 (\theta_1 < \theta_2)$ against $H_1: \theta_1 < \theta < \theta_2$ that is of the form

$$\varphi(x) = \begin{cases} 1 & \text{if} \quad c_1 < T(x) < c_2, \\ \gamma_i & \text{if} \quad T(x) = c_i,\ i = 1, 2\ (c_1 < c_2), \\ 0 & \text{if} \quad T(x) < c_1 \text{ or } > c_2, \end{cases}$$

where the c's and γ_i's are given by $E_{\theta_1}\varphi(X) = E_{\theta_2}\varphi(X) = \alpha$

(e) Obtain UMPU test of size α for testing $H_0:\theta = \theta_0$ vs $H_1:\theta \neq \theta_0$ in (k+1) - parameter EFD using ancillary statistic.

(f) Let $X_1, \dots, X_n$ be a r.s. from $U(\theta_1, \theta_2)$ distribution. Obtain an UMP test for testing $H:\theta_1 \leq 0$ vs $K:\theta_1 < 0$ using ancillary statistic.

(g) For the SPRT with stopping bounds (A, B) and strength (α, β) show that

$$A \leq \frac{1-\beta}{\alpha} \quad \text{and} \quad B \geq \frac{\beta}{1-\alpha}$$

(h) Let $\mathcal{F}$ denote the class of all distributions with finite first moment μ. Check whether $\mu^3(F)$ is estimable or not. If estimable then Find : (i) degree m, (ii) kernel, (iii) symmetric kernel and (iv) U-statistic.

(i) Differentiate between parametric and non parametric tests.

3 (a) Define MLR of a distribution. Show that a UMP test for composite null vs composite alternative exists for the distributions involving single parameter and have MLR property. **06**

(b) Let $X_1, \dots, X_n$ be a r.s. from $N(\theta, \sigma^2)$ distribution. Using ancillary statistic construct UMPU test of size α for testing $H:\theta = \theta_0$ vs $K:\theta \neq \theta_0$. Hence construct $100(1-\alpha)\%$ UMA unbiased CI for θ. **06**

OR

(b) A UMP unbiased test for testing $H:\theta = \theta_0$ vs $K:\theta \neq \theta_0$ in an one-parameter EFD exists and is of the form (*stated by you*) if and only if $E_{\theta_0}\varphi(X) = \alpha$ and $E_{\theta_0}T\varphi(X) = \alpha E_{\theta}T$.

4 (a) Define Likelihood Ratio Test (LRT). Specifying the under lying assumptions, obtain the **06**

asymptotic distribution of $-2log\lambda$.

(b) What are the weaknesses of LRT. Specifying the under lying assumptions show that LRT is a consistent test. **06**

OR

(b) Let X be a binomial $b(n,p)$ random variable. Develop the level α LRT of $H_0: p \leq p_0$ against $H_1: p > p_0$ and construct $100(1-\alpha)\%$ for p.

5 (a) If $Z = \log\{f(x;\ \theta_1)/f(x;\ \theta_0)\}$ is a r.v. such that **06**

$$P\{Z > 0\} > 0 \text{ and } P\{Z < 0\} > 0,$$

$$M(t) = E(e^{tZ}) \text{ exist for any real value t,}$$

and $E(Z) \neq 0$, then show that there exist a $t^* \neq 0$ such that $M(t^*) = 1$.

Further, using this result, obtain the approximate OC function of the test for testing simple null hypothesis versus simple alternative hypothesis.

(b) State and prove Wald's fundamental inequality. Discuss its special cases. **06**

OR

(b) Discuss SPRT for testing $H: \theta = \theta_0$ vs $K: \theta = \theta_1$ when $X \sim N(\theta; 1)$. Also, obtain its OC function when $K: \theta \leq \theta_1$.

6 (a) Define one-sample U-statistic for an estimating parameter γ of cdf F(x). Obtain its mean and variance **06**

(b) Obtain asymptotic distribution of one-sample U-statistic. **06**

OR

(b) Discuss at least two tests which based on U-statistics.

(3)