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SARDAR PATEL UNIVERSITY
M.Sc. (II Semester) Examination
2019
Monday, 18th March
10.00 am to 1.00 pm
STATISTICS COURSE No. PS02CSTA01/PS02CSTA21
(Stochastic Processes)

Note: Figures to the right indicate full marks of the questions. **(Total Marks: 70)**

1 Attempt all, write correct answers **08**

(i) The average time to absorb for Markov Chain having tpm $\begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$ is ______

a) 1
b) more than 1
c) less than 1
d) 0

(ii) The period of an Ehrenfest chain is ____.

a) 1
b) 2
c) more than 2
d) 0

(iii) Poisson process is additive but not ____________

a) covariance stationary
b) decomposable
c) generalized
d) subtractive

(iv) The probability generating function having argument *s* of the symmetric geometric branching process for 2nd generation is given by which of the following?

a) 1/(2-*s*)
b) (2-*s*)
c) (2-*s*)/(3-2*s*)
d) (3-2*s*)

(v) Pure birth process is a version of Poisson process in which the rate of events are __________

a) Time homogeneous
b) State homogeneous
c) State dependent
d) Time nonhomogeneous

(vi) The mean of a linear birth and death process having equal birth and death rate is 1 if N(0) = __.

a) 1
b) i>1
c) 0
d) i<1

(vii) In usual notation, the Wiener process is analogous of _______ and the Ornstein-Uhlenbeck process is analogous of _______.

a) *AR(1), RWM*
b) *AR(2), RWM*
c) *RWM, AR(1)*
d) None of these

(viii) The steady state diffusion parameter of Ornstein-Uhlenbeck process in usual notaion is given by _____

a) $\sigma^2/4$
b) $\sigma^2/2$
c) $2\sigma^2$
d) $\sigma^2/2\beta$

2 Attempt ANY 7, each carries 2 marks **14**

(a) Characterize the Markov chain having TPM $P = \begin{bmatrix} 1 & 0 & 0 \\ .3 & .4 & .3 \\ .2 & .1 & .7 \end{bmatrix}$ in three properties.

(b) Define time reversible Markov Chain. And narrate one of the associated results.

(P.T.O.)

(c) What is random walk model? Verify when it is recurrent.

(d) Justify that Poisson process is a special case of non-homogeneous Poisson process.

(e) Suppose the particles arrive at a Geiger counter according to a PP(λ) with λ=1000/sec. However, the counter fails to count a particle with probability .1 independent of everything else. Suppose the counter registers 4 particles in .02 seconds. What is the probability that at least 6 particles must have actually arrived?

(f) Derive P [N(t) =1] for a pure birth process $\{N(t), t \geq 0\}$.

(g) Write down difference – differential equation for birth and death process under linearity conditions $\lambda_n = \lambda$, $\mu_n = n\mu$. Then transform into the pgf form.

(h) State appropriateness of Wiener process and the O-U process for modelling the Brownian motion.

(i) Give an application for each, the Markov chain, Poisson process, birth and death process and the O-U process.

3(a) Define Ehrenfest chain and Gambler's ruin chain. Also characterize these two for Markov Chain properties. **06**

3(b) Compute limiting probability model for the following transition matrix. **06**

$$\begin{bmatrix} .3 & .7 & 0 & 0 \\ .4 & .6 & 0 & 0 \\ .2 & .3 & .5 & 0 \\ .5 & 0 & .4 & .1 \end{bmatrix}$$

OR

Consider a DTMC on state space {1, 2, 3, 4} having tpm P as given here 3(b) Suppose $X_0 = 1$ with probability 1. Compute $E(X_1 = 4)$, $P(X_1=2, X_2=4, X_3=1)$.

4(a) Derive mean and variance of compound Poisson process. **06**

4(b) Arguing according to the definition of Poisson process compute the probability. Suppose customer arrive at a post office according to a Poisson process with rate 10/hour. What is the probability that one customer arrives between 1 pm and 1.06 pm, and two customers arrive between 1.03 pm and 1.12 pm? **06**

OR

State and prove Campbell's theorem.

5(a) Show that for linear death process with μ denoting death rate, N(0) = n its probability model is Bin(n, $exp(-\mu t)$, $t \geq 0$). **06**

5(b) Describe that the simple trunk line process is a birth and death process under linearity conditions $\lambda_n = \lambda$, $\mu_n = n\mu$. Obtain its steady state as well as the general probability models. **06**

OR

Derive mean of linear birth and death process. What is its steady-state value?

6(a) Derive the Kolmogorov's forward differential equation for Wiener process. **06**

6(b) Write down Kolmogorov's forward differential equation for Wiener process and show that standard normal transition density $f(t, x)$ with initial position $x_0 = 0$ satisfies it. **06**

OR

Write about the Ornstein-Uhlenbeck process.

—X—