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SEAT No. \_\_\_\_\_

**SARDAR PATEL UNIVERSITY**  
**M.Sc. (Statistics) Fourth Semester and Certificate**  
**Course in Financial Statistics Examinations**

Saturday, October 27, 2018

02:00 p.m. to 05:00 p.m.

**STATISTICS COURSE :PS04ESTA02(Actuarial Statistics)**

Note: Figure to the right indicate full marks to the questions.

**Total Marks 70**

1. Choose the correct answer(s) and write in your answer book. 8
- (i) Pure risk has outcomes:
- (a) either a loss or no loss (b) only loss  
(c) loss, gain and no change (d) none of (a) to (c)
- (ii) Let  $\mu_x$  and  $S(x)$  be mortality rate and survival function of a random variable  $X$  with pdf  $f(x)$ . Then which of followings is most correct.
- (a)  $f(x) = \mu_x S(x)$  (b)  $f(x) = \mu_x / S(x)$   
(c)  $f(x) = S(x) / \mu_x$  (d) None of (a) to (c)
- (iii) Which of the following is **not** a survival function for  $x \geq 0$
- (a)  $S(x) = \exp(x - 0.7(2^x - 1))$  (b)  $S(x) = \exp(-x^2)$   
(c)  $S(x) = \frac{1}{(1+x)^2}$  (d)  $S(x) = \exp(-x)$
- (iv) Let  $e_x$  be mean of curtate future life random variable of life  $x$ .  
Which of the following is correct?
- (a)  $e_x = \sum_{k=1}^{\infty} k p_x$  (b)  $e_x = p_x(1 + e_{x+1})$   
(c) Both (a) and (b) (d) None of (a) to (c)
- (v) Which of the following is a utility function
- (a)  $U(w) = -e^{aw}, 0 < w < \infty, a > 0$  (b)  $U(w) = -w^{-\gamma}, w > 0, 0 < \gamma < 1$   
(c)  $U(w) = -w, 0 < w < \infty$  (d)  $U(w) = -aw - b, w > 0; a, b > 0$
- (vi) Which of the following is not correct?
- (a) Density function of  $T(x)$  is  $\mu_{x+t} {}_t p_x$  (b)  $\frac{d}{dt} l_{x+t} = -l_{x+t} \mu_{x+t}$   
(c)  $L_x = \int_0^1 l_{x+t} dt$  (d)  $e_x = E(T(x))$

(1)

(PTO)

(vii) The relation between age specific death rate  $m_x$  and  $q_x$  is given by

$$(a) q_x = \frac{m_x}{\left(1 + \frac{m_x}{2}\right)}$$

$$(b) m_x = \frac{q_x}{\left(1 + \frac{q_x}{2}\right)}$$

$$(c) q_x = m_x \left(1 + \frac{m_x}{2}\right)$$

$$(d) m_x = q_x \left(1 + \frac{q_x}{2}\right)$$

(viii) Consider the schemes where benefits are paid at the end of year of death.

The notation  ${}_m|A_{x:\overline{n}|}^1$  stands for the net single premium of

(a) whole life insurance scheme

(b) n-year term insurance scheme

(c) m year deferred n year endowment insurance scheme

(d) m year deferred n year term insurance scheme

2. Answer any **seven** of the following.

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- (a) Describe financial risks involved with an individual. What types financial losses insurance companies try to compensate and how?
- (b) Describe collective risk model.
- (c) Explain curtate future life time random variable and obtain its probability function when life time follows exponential distribution.
- (d) Obtain the probability density function of  $T(x)$ , future life time, when life time is modeled by Gompertz's force of mortality
- (e) Suppose life length random  $X$  has a distribution with survival function  $S(x) = 1 - \left(\frac{x}{w}\right)^5, 0 < x < w$ . Find  ${}_{2|2}q_4$ .
- (f) Find the distribution of curtate future life time random variable, when the life length random variable is modeled by Makeham's Law, at life time  $x=30$
- (g) Obtain a relation between  $l_x$  and  $\mu_x$
- (h) Prove that  $a_{\overline{n}|}^{(m)} < \bar{a}_{\overline{n}|} < \ddot{a}_{\overline{n}|}^{(m)}$
- (i) An amount Rs.10000 is payable on December 31, for 10 years. The first payment is due in 2010. Find the purchase price of the annuity on January 1, 2000 with 6% annual effective rate of interest.

3 (a) Discuss in detail about the construction of life table and their uses in insurance business.

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- (b) Suppose life length random variable X has a distribution with survival function

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$$S(x) = \begin{cases} 1 - \frac{x^k}{10^k} & \text{if } 0 \leq x \leq 10, \text{ where } k > 0. \\ 0 & \text{Otherwise} \end{cases}$$

Obtain (i) force of mortality (ii)  ${}_t p_4$  (iii) density function of T(4).

OR

- (b) A life insurance company issues 1-year term life contract with benefit amounts b equal to 3 and 4 to individuals with probabilities of death 0.05 and 0.07 respectively. The following table specifies the number n in each class.

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k	$q_k$	$b_k$	$n_k$
1	0.05	3	300
2	0.05	4	200
3	0.07	3	500
4	0.07	4	200

Company wants to collect an amount equal to 95<sup>th</sup> percentile of the distribution of claims with each individuals share to be  $(1+\theta)E(X)$ . Calculate  $\theta$ .

- 4 (a) Write in detail about different types of annuities you have studied in the course.
- (b) Discuss in detail n year term insurance when benefits are payable at the moment of death. Obtain the expression for net single premium when life time is modeled by

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$$S(x) = \begin{cases} 1 - \left(\frac{x}{100}\right)^k & \text{if } 0 \leq x \leq 100, \text{ where } k > 0 \\ 0 & \text{Otherwise} \end{cases}$$

OR

- (b) If  $l_x = 1000\sqrt{(100-x)}$ ,  $0 \leq x \leq 100$ , calculate exact value of  $\mu_{36+\frac{1}{4}}$ . Also find it under the assumption of uniformity of deaths in a unit interval. Compare the two values.

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- 5 (a) Write a detailed note on insurance business in India
- (b) Write in detail about n year pure endowment insurance when benefit are payable at the moment of death and obtain expression for the net single premium when life distribution is given by

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(3)

(PTO)

$$S(x) = \begin{cases} 1 - \frac{x^k}{100^k} & \text{if } 0 \leq x \leq 100, \text{ where } k > 0. \\ 0 & \text{Otherwise} \end{cases}$$

OR

- (b) Suppose life time random variable is modeled by a uniform distribution over (0,100). Find net single premium for a whole life insurance policy of rupees one lakh, when benefit are payable at the moment of death, for ages  $x = 25, 35$  and  $45$ . 6
- 6 (a) Write detailed note on different type of annuities. 6
- (b) A loan of Rs.100000/- is taken on January 1, 2015. It has to be repaid in 5 equal installments payable yearly at the beginning of the year. Based on 6.5% annual rate of interest determine the amount of installment. 6

OR

- (b) Describe n-year temporary life annuity. Suppose the force of mortality follows truncated exponential over (0,100) Find the value of  $\bar{a}_{25:\overline{5}|}$  6

