(120/121) SEAT No.________ Number of Printed Pages 4

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SARDAR PATEL UNIVERSITY
M.Sc.(Statistics) First Semester Examinations
Monday, October 29, 2018
Time 10:00 a.m. to 1:00 p.m.
Subject:PS01CSTA24/PS01CST04:Statistical Inference I

Note: (i) Figures to the right of questions indicate maximum marks.

(ii) Total Marks is 70

1 Choose the most correct answer(s) and write in your answer book. **8**

(i) Let $X_1, X_2, \dots, X_n$ be random sample on X which follows B(1,θ) distribution. Which of the following is correct?

(a) $(X_{(1)}, X_{(2)}, \dots, X_{(n)})$ is the only vector sufficient statistic for θ.

(b) Only $(X_{(1)}, X_{(2)}, \dots, X_{(n)})$ and $\sum_{i=1}^{n} X_i$ are sufficient for θ

(c) $(X_{(1)}, X_{(2)}, \dots, X_{(n)})$ is sufficient and $\sum_{i=1}^{n} X_i$ is minimal sufficient

(d) $\sum_{i=1}^{n} X_i$ is the sufficient statistics for θ

(ii) Let $X_1, X_2, \dots, X_n$ be random sample on X which follow Poisson distribution with mean λ. Which of the following correct?

(a) The conditional distribution $X_1 | \sum_{i=1}^{n} X_i$ is free from λ.

(b) $\sum_{i=1}^{n} X_i$ is minimal sufficient for λ.

(c) $\sum_{i=1}^{n} X_i/n$ is unbiased for λ

(d) All (a), (b) and (c) are correct.

(iii) An estimator is $T_n$consistent for θ. Then which of the following are correct?

(a) $T_n \xrightarrow{WP1} \theta$

(b) $T_n \xrightarrow{P} \theta$

(c) Mean of T_n converges to θ and vanraince of T_n tends to zero

(d) Both (b) and (c)

(iv) Suppose Variance of unbiased estimator T_n of θ attains C-R lower bound. Then which one of the following correct?

(a) T_n is UMVUE of θ.

(b) T_n is unique UMVUE of θ.

(c) T_n is Minimum Variance Bound Estimator.

(d) None of (a) to (c).

(P.T.O)

(v) Which of the following is true for moment estimator?

(a) Moment estimators of θ is always unbiased.
(b) Moment estimators is consistent .
(c) Moment estimator is also mle.
(d) Moment estimators is CAN

(vi) MLE of lower truncation parameter θ of a lower truncation family of distributions based on a random sample of size n is given by

(a) Median
(b) Mean
(c) Sample minimum
(d) Sample maximum

(vii) Prior proportional to square root of Fisher Information is called

(a) Jefrey Prior
(b) Conjugate prior
(c) Non-informative prior
(d) Baye's prior

(viii) Which of the following does not satisfy C-R regularity conditions

(a) One parameter exponential family of distributions
(b) Normal distribution with variance 1.
(c) Pitman family of distributions.
(d) Geometric distribution

2 Answer any **seven** of the following.(Short Answer type) **14**

(a) Obtain sufficient statistic for the μ based on n i.i.d observations from the density $f(x, \mu)= \exp(x- \mu)$, $x > \mu$.

(b) Define score function and show that it is unbiased for 0. Also obtain its variance and comment on it.

(c) Show that sample maximum is the m.l.e of θ when the random sample of size n is drawn from U(3,θ).

(d) Show that Cauchy distribution with location parameter μ and scale parameter θ does not belong exponential family of distributions.

(e) Show that the family of normal distributions $\{N(\mu, 1): -\infty < \mu < \infty\}$ is complete.

(f) Obtain Fisher information contained in the sufficient statistic about θ, based on a random sample of size n from N(2, θ).

(g) Let $f(x,\theta) = \theta(1-\theta)^x$, x=0,1,2,3,... be the probability distribution X . Based on a random sample size n obtain moment estimator of θ. Is it exactly unbiased or asymptotical unbiased?

(h) Discuss need for C-R lower bound.

(i) Let $X_1, X_2, and\ X_3$ be i.i.d Poisson with mean λ. Obtain conditional distribution of $X_1|T$ where T $=X_1 + X_2 + X_3$ and comment on the result.

3. (a) State and prove Rao-Blackwell-Lehman- Scheffe theorem by giving in detail all necessary regularity conditions. Give an example of sufficient statistic which is not complete. 6

(b) Describe one parameter exponential family of distribution by clearly giving all the regularity conditions. State Fisher-Neyman Factorization theorem and use it to obtain sufficient statistic for the family. How does it help to obtain sufficient statistic for σ^2 in $N(0,\sigma^2)$? 6

OR

(b) Explain with the help of example how sufficient statistic partitions the sample space. Define minimal sufficient statistics. With the help of likelihood equivalence principle obtain minimal sufficient statistics for probability of success θ based on random sample from geometric distribution. 6

4 (a) State and Prove Cramer-Rao inequality by giving in detail all regularity conditions. 6

(b) Let $f(x,\theta) = \begin{cases} \frac{a(x)(h(\theta))^{d(x)}}{g(\theta)} & if\ x \in \mathcal{X} \\ 0 & Otherswise \end{cases}$. Cleary stating necessary conditions on the functions involved in the probability function, explain Roy-Mitra Technique to obtain UMVUE of the parametric function $\psi(\theta) = (h(\theta))^r (g(\theta))^s$ whenever it is estimable. 6

OR

(b) Obtain unbiased estimators based on a random sample of size n for (i) mean of U(0,θ) and (ii) parametric function $\psi(\theta) = \theta^k$, where $k \geq 1$ and θ is the mean of exponential distribution. Obtain Cramer-Rao lower bound for the variance of these unbiased estimators, if they exist. In case does not exist, give the reason. 6

5 (a) State and Prove Bhattacharyya System of lower bound. Give all necessary regularity conditions. 6

(b) Describe maximum likelihood method of estimation. Obtain maximum likelihood estimator (mle) of the parameters based on a random sample size n from (i) $U(0,\theta)$ (ii) exponential distribution with pdf $f(x,\mu) = \begin{cases} \frac{e^{-(x-\mu)/4}}{4} & if\ x > \mu \\ 0\ otherwise. \end{cases}$. 6

OR

(b) Show that moment estimator, based on random sample of size n, of the parametric function $\psi(\theta) = \theta^k$ is consistent; here ≥ 1 , and θ is the mean of exponential distribution. Obtain moment estimator, based on random sample of size n, of the parameter θ from U(0,θ). Is it same as mle of θ? 6

CP.T.O)

6 (a) Show that Bayes' estimator of a scalar parameter θ under the squared error loss function is its posterior mean. Use the result and obtain Bayes estimator, based on a random sample of size n, of mean θ of B(1,θ) under the prior distribution $\pi(\theta) = c\sqrt{\frac{1}{\theta(1-\theta)}}$. 6

(b) Let X follows Poisson distribution with parameter λ. Based on random sample of size n, obtain the mle of λ and show that it is CAN estimator. Is it BAN? 6

OR

(b) Describe method of scoring for estimating θ in one parameter probability function f(x,θ) based on a random sample size n. Discuss how you will use the method to obtain mle of the following distribution. 6

$$f(x,\theta) = \frac{1}{\pi[1+(x-\theta)^2]}, -\infty < x, \theta < \infty$$

—X—

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