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SARDAR PATEL UNIVERSITY
M.Sc. Statistics (Ist semester) Examination
2018
Monday, 22^{nd} October
10.00 am to 1.00 pm
Course No. PS01CSTA21
(Probability Theory)

Note: Figures to the right indicate marks. (Total marks: 70)

Q1 Multiple Choice Questions. Write correct answer. 08

I) For a class C={ $(a,b]$, $0<a\leq b<\infty$ ________ } to be a semi-field, the missing elements is given by ____

a) $(-\infty, a] \cup (b, \infty)$ b) $(-\infty, a]$, (b, ∞), $\Re^+$
c) $(0, a]$, (b, ∞), $\Re^+$ d) $(0, a]$, (b, ∞)

II) Lebesgue measure of set of rational numbers is
a) 0 b) $F(x)$-$F(x$-)
c) ∞ d) $\sum_Q F(x)$-$F(x$-)

III) The probability measure of event E based on the Lebesgue measure is given by
a) $\lambda(\Omega)$ b) $\lambda(E)$
c) $\mu(\Omega)/\ \mu(E)$ d) $\lambda(E)/\lambda(\Omega)$

IV) The expectation of the random variable $I(A^c)$ is

a) $P(A)$ b) 1-$P(A)$
c) 0 d) $P(A)$ - 1

V) Let $A_n = [1, 1+1/n)$, $n\geq 1$. Then limit set is ____
a) $\cap A_n$ b) $\cup A_n$
c) *None of these* d) ϕ

VI) The characteristic function of X/σ where X is a normal random variable with mean 0 is _____

a) 1 b) $\exp(-t^2/2)$
c) $\exp(-t^2\sigma^2/2)$ d) $\exp(-t^2/2\sigma^2)$

VII) For which of the following the monotone convergence theorem will hold?
a) $\sqrt{X_n}$, $n\geq 1$ b) $\sqrt{n}X$, $n\geq 1$

c) $\sqrt{n}X^2$, $n\geq 1$ d) any sequence of rvs

VIII) Number of discontinuity points of a Bernouli distribution function is
a) 0 b) 1
c) 2 d) ∞

Q2 Attempt any 7. Each carries 2 marks. 14

a) Obtain semi-fields defined on (i) {1, 3, 5, 7, 9, 11} (ii) $(a, b]$, $a< b \in \Re$.

(P.T.O.)

b) Show that probability measure is sub additive.

c) Find the limit of sequence of sets $\{A_n, n\geq 1\}$ defined as $A_n = (1-\frac{1}{n}, 2+\frac{1}{n})$ if n is odd and $A_n = (\frac{1}{n}, 1+\frac{1}{n})$ if n is even.

d) Evaluate $\int X d\mu_F$ where $F(x) = \begin{cases} 0 \; if \; x < 0 \\ \frac{1}{2} if \; x = 0 \\ \frac{2}{3} \; if \; 0 < x \leq 3 \\ 1 \; otherwise \end{cases}$

Where X defined on $\Omega = (-1, 5)$ is taking values $X(\omega)=½$, $\omega \in (-1,2)$, $X(\omega)=1/3$, $\omega \in [2,2]$, $X(\omega) =1$, $\omega \in (2,3]$ and $X(\omega)=0$, $\omega \in (3,5)$.

e) If $\varphi(t)$ is a characteristic function of random variable X, then obtain expression for $\varphi'(t)$.

f) State the conditions under which the Basic inequality becomes Markov's inequality and Chebychev's equality.

g) Show that every distribution function has at most countable discontinuities.

h) Explain the uses of Laws of Large Numbers?

i) State Liaponouv's Central Limit Theorem (CLT). How it is different from Lindberg-Levy CLT.

Q3 a) Define field and sigma field. State the inter relationship between two classes. Show that intersection of sigma fields is also a sigma field. 6

b) Define all classes of subsets of $\Re$, the real line. Mention, which one of these is the Borel field and the Borel sigma field? 6

OR

b) Obtain minimal sigma field containing $\{(a, b], (c, d]\}$, $a<b$, $c<d$ real constants.

Q4 a) Define Probability function. State all its properties. Prove that probability function is continuous. 6

b) Show that if X is a non-negative random variable then there exist a non-decreasing sequence of non-negative simple random variables that converges to X. 6

OR

b) Define non-negative random variable. Use it to show that, if X and Y are two random variables then $X+Y$ is also a random variable. What if the arithmetic operation is multiplication? 6

Q5 a) State and prove the Fatou's theorem. Then use it in proving the Dominated Convergence Theorem. 6

b) State and prove a moment's inequality you know. Also state its uses. 6

OR

b) State Markov's inequality. If X is a random variable taking values $-a$, 0, a with probabilities p, $1-2p$, p respectively, show that Markov's inequality attains equality. 6

Q6 a) State the sufficient conditions for $\{X_n, n \geq 1\}$ to hold each, the weak law of large numbers, the strong law of large numbers and the central limit theorem. Use the appropriate one to check whether sequence having pmf hold law of large numbers or not. 6

$$P(X_n = n) = \frac{1}{2n} = P(X_n = -n) \quad P(X_n = 0) = 1 - \frac{1}{n}$$

b) State and prove Kintchin's weak law of large numbers (WLLN), stating the modes of convergences of sequence of random variables used in proving the law. 6

OR

b) If X_i's are independent standard normal variables, using appropriate result find the distribution function of $\sum_{i=1}^{n} X_i$. 6

— X —

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