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[133] SEAT No.________

No. of printed pages: 03

SARDAR PATEL UNIVERSITY
M.Sc. (Statistics) 1st Semester Examination
2018
Friday, 26th October
10:00 a.m. to 1:00 p.m.
Course No. PS01CSTA03/PS01CSTA23
(Distribution Theory)

Note: Figures to the right indicate marks. (Total marks: 70)

1 **Write the correct answer (each question carries one mark).** **08**

(a) Let X and Y be independent $N(\mu, 1)$ r.vs. What will be the distribution of $(X+Y)^2/2$?

(A) Non-central χ^2 distribution
(B) Non-central t distribution
(C) Non-central F distribution
(D) Normal distribution

(b) The distribution $g(\theta)f(x/\theta)$ is called

(A) mixture distribution
(B) compound distribution
(C) contagious distribution
(D) absolute distribution

(c) The conditional mean of $(\underline{X}_{(1)}|\underline{x}_2)$ is known as

(A) regression
(B) linear regression
(C) multiple linear regression
(D) none

(d) The multinomial distribution is non-singular if

(A) covariance matrix Σ is non-singular
(B) Σ is positive definite
(C) Σ is singular
(D) (A) & (B) but not (C)

(e) Let X_1 and X_2 be two iid $N_p(\underline{0}, I)$ r.vs. Suppose A is a $p \times p$ symmestric idempotent matrix of rank r. Which of the following statement is true?

(A) $X_1'AX_2 \sim \chi_r^2$
(B) $X_1'AX_1 + X_2'AX_2 \sim \chi_{2r}^2$
(C) $X_1'AX_1 + X_2'AX_2 \sim 2\chi_r^2$
(D) none of the above

(f) The formula for partial correlation coefficient in terms of inverse elements of covariance matrix $\Sigma^{-1} = (\sigma^{ij})$ is

(A) $-\sigma^{12}/\sqrt{\sigma^{11}\sigma^{22}}$

(PTO)

(B) $\sigma^{12}/\sqrt{\sigma^{11}\sigma^{22}}$
(C) $-\sigma^{12}/\sigma^{11}\sigma^{22}$
(D) $\sigma^{12}/\sigma^{11}\sigma^{22}$

(g) Let X and Y be two independent r.vs with common cdf F and let $Z = \min(X, Y)$. Then the CDF of Z is
(A) $[1 - F(z)]^2$
(B) $[F(z)]^2$
(C) $1 - [1 - F(z)]^2$
(D) $1 - [F(z)]^2$

(h) Let $X_1, \ldots, X_m$ be iid r. vs. with cdf $F(x)$, $Y_1, \ldots, Y_n$ be iid r. vs. with cdf $G(x)$ and $X's$ and $Y's$ be independently distributed. For testing $H_o: F(x) = G(x)\ \forall\ x$, which of the following test is used?
(A) Wilcoxon signed rank test
(B) Kolmogorov-Smirnov test
(C) Wald-Wolfowitz run test
(D) none of the above

2 Answer any SEVEN of the following. **14**

(a) Let X_1, X_2, X_3 be iid $N(0,1)$ variables. Find the distribution of
$$Y = X_1^2/(X_1^2 + X_2^2 + X_3^2)$$

(b) Define mixture distribution. Suppose $f(x,\theta) = \dfrac{e^{-\theta}\theta^x}{x!}$ for $x = 0, 1, 2, \ldots$ and $g(\theta) = \dfrac{\lambda^r}{\Gamma(r)}\theta^{r-1}e^{\lambda\theta}, \theta > 0$. Show that the mixture distribution is negative binomial distribution with parameters r and $p = \lambda/(\lambda+1)$.

(c) What is regression? Show that it is conditional mean.

(d) Show that the pmf of non-singular multinomial distribution is the general term of the series expansion
$$\left(1 + \sum_{i=1}^{k}\theta_i\right)^n = \sum\cdots\sum \frac{n!}{(n - \sum x_i)!}\prod_{i=1}^{k}\left(\frac{\theta_i^{x_i}}{x_i!}\right)$$

(e) Define sample range and obtain its density.

(f) Define rank statistics obtain their distribution and discuss Wilcoxon signed rank statistic.

(g) Consider sampling from the exponential distribution. Obtain the limiting distribution of $\left(X_{(n)}^n - a_n\right)/b_n$ for suitably chosen constants a_n and b_n.

(h) Let $\underline{X} \sim N_3\left(\underline{\mu}, \Sigma\right)$ where $\underline{\mu}' = (2,1,1)$ and $\Sigma = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. Obtain the joint distribution of $Y_1 = X_1 + X_2 + X_3$ and $Y_2 = X_1 - X_2$.

(i) Let $\underline{X} \sim N_p\left(\underline{\mu}, \Sigma\right)$, Σ is p.d., and let A be a symmetric matrix. Assuming that $\underline{X}'A\underline{X}$ is distributed as non-central χ^2 distribution find its mean.

3 (a) Define non-central t distribution and obtain its density. Also, obtain the mean. **06**

(b) Obtain mgf of a non-central chi-square distribution. Using this mgf (i) obtain mean the distribution, (ii) obtain mgf of central χ^2 and (iii) prove additive property. **06**

OR

(b) Define non-central F variable and derive its density and first moment.

4 (a) Define multiple correlation coefficient and derive its formulae. Also, derive it for multinomial distribution. **06**

(b) Define multinomial distribution. Obtain variance-covariance matrix of this distribution. Check whether it is p.d. or p.s.d. **06**

OR

(b) Discuss: Transformation of statistics and its roll.

5 (a) Let $X_1, \ldots, X_n$ be a r.s. drawn from continuous cdf $F(x)$ and $X_{(1)}, \ldots, X_{(n)}$ be the corresponding order statistics. Obtain the marginal distribution of $X_{(i)}$. In particular for $U(0,\theta), \theta > 0$, distribution, for n = 3, if $E(T) = \theta$, where $T = k[12X_{(1)} + 6X_{(2)} + 4X_{(3)}]$, determine the constant k. **06**

(b) State and prove "Probability Integral Transformation". Obtain the joint distribution of any two transformed variables and hence the area under the density between two ordered observations, **06**

OR

(b) Write a note on "extreme value statistics and their asymptotic distributions".

6 (a) State and prove the Cochran theorem on distribution of quadratic form. **06**

(b) Define multivariate normal distribution. Obtain its mgf. Using this mgf establish any two properties. **06**

OR

(b) Let $\underline{X}$ be distributed $N_3(\underline{\mu}, \Sigma)$ with the pdf $f(\underline{x}) = k \cdot \exp\{-Q/2\}$

where

$$Q = \frac{3}{2}X_1^2 + 2X_2^2 + X_3^2 - 3X_1X_2 - 2X_1X_3 - 2X_2X_3 + 10X_1 - 14X_2 + 8X_3 + 26$$

Find (i) the normalizing constant (ii) mean vector (iii) covariance matrix.

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