[124/125] SEAT No.________ **No. of Printed Pages : 4**

SARDAR PATEL UNIVERSITY

M.Sc External Examination, Semester - I

Wednesday Date : 24-10-2018

Time : 10.00 am to 1.00 pm

Subject/Course Code : PSO1CSTA02/22

Matrix Algebra

Q-1 Attempt following 08

1 In usual notation orthogonal compliment of matrix A is denoted by

a A'

b A^{-}

c A^{+}

d $A^{\perp}$

2 In usual notation for idempotent matrix A has property

a $A = A'$.

b $A = A^2$.

c $A = A^3$.

d Non-of-above.

3 In usual notation latent roots of matrix A is 1 or 0 if

a Idempotent.

b Singular.

c Non-singular.

d Non-of-above.

4 In usual notation length of vector for $\underline{X}' = [1 \quad 2 \quad 3]$

a 14.

b $\sqrt{14}$.

c $1/\sqrt{14}$

d Non-of-above.

5 In usual notation λ denotes latent roots of matrix A then latent roots of $C^{-1}AC$ is

(P.T.O.)

a $c\lambda$.

b $c\lambda^{-1}$.

c $c^{-1}\lambda$.

d Non-of-above.

6 In usual notation for square matrix A and B of order n, the rank of matrix AB; r_{AB}

a $r_{AB} \geq r_A + r_B - n$.

b $r_{AB} = r_A + r_B - n$.

c $r_{AB} \leq r_A + r_B - n$..

d Non-of-above.

7 In usual notation $J_{r\times s} J_{s\times t}$ is

a $J_{r\times s} J_{s\times t} = rJ_{r\times t}$.

b $J_{r\times s} J_{s\times t} = rsJ_{r\times t}$.

* c $J_{r\times s} J_{s\times t} = sJ_{r\times t}$.

d Non-of-above..

8 In usual notation for centering matrix; C and $J = \underline{1}\,\underline{1}'$

a $CJ = JC \geq 0$

b $CJ = JC = 0$

c $CJ = JC > 0$.

d Non-of-above.

Q-2 Attempt any **SEVEN** 14

(1) In usual notation define centering matrix C_n.

(2) In usual notation give three interpret of $Dim(\tau) = 11$.

(3) Prove : For symmetric matrix, latent vectors corresponding to distinct latent roots are $\perp$.

(4) Evaluate $\underline{1}'\underline{X}$, when $\underline{X}' = [1, 2, 3]$.

(5) In usual notation react and justify : $\underline{\hat{X}} = (I - H)\underline{Z}$ is solution vectors of $A\underline{X} = \underline{b} \neq \underline{0}$.

(6) In usual notation A^{-} is g-inverse of matrix A if $A = AA^{-}A$. Define g-inverse of A.

(7) In usual notation : $S = X'X, H = S^{-}S$. Write X^{-}; g-inverse of X and show that $XX^{-}X = X$.

(8) In usual notation react and justify : for centering matrix of order n has latent root $\lambda = 1$ with multiplicity $m = (n - 1)$.

(9) In usual notation mean vector and variance covariance matrix of random $\underline{X}$ is $E(\underline{X}) = \underline{\mu}$ and $\Sigma = V(\underline{X})$. Derive $E(\underline{X}\,\underline{X}')$ for both cases $\underline{\mu} = \underline{0}$ and $\underline{\mu} \neq \underline{0}$.

Q-3 A In usual notation react and justify : for idempotent matrix M_{nXn} has property $r_{(I - M)} \geq n - r_M$. 06

Q-3 B In usual notation show that $XS^{-}X'$ is invariant for any choice of S^{-}, g-inverse of $S = X'X$. 06

OR

Q-3 B In usual notation derive LIN solution vectors for $A\underline{X} = \underline{0}$, where $A_{nXm}, r_A = k$. 06

Q-4 A In usual notation explain : Equation of model. Derive solution for normal equation for full rank and non-full rank model.

Q-4 B State and prove diagnability theorem. 06

OR

B In usual notation state and prove full rank factorization theorem. 06

OR

Q-5 A In usual notation show that : $tr(A) = \sum_{i=1}^{n} \lambda_i$ and $\|A\| = \prod_{i=1}^{n} \lambda_i$. 06

Q-5 B State two definition of g-inverse of matrix and establish equivalence between them. 06

OR

Q-5 B State and prove necessary and sufficient condition for matrix A to be positive definite. 06

Q-6 A In usual notation prove : The $\perp^{er}$ $\underline{\gamma}$ from $\underline{\alpha}$ to $\Im$ has property $\| \underline{\gamma} \| = \inf \| \underline{\alpha} - \underline{X} \|$ for $\forall \underline{X} \in \Im$,

Q-6 B In usual notation show that $r_{AB} \leq r_A \; and \; r_B$. 06

OR

Q-6 B Consider system of non-homogeneous equations, $A\underline{X} = \underline{b}$. In usual notation show that $\|e(\underline{X})\| \geq \underline{b}'(H - I)\underline{b}$. 06

— X —

(4)