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SARDAR PATEL UNIVERSITY

M.Sc.(Statistics) First Semester Examination

Thursday, December 6, 2012

Time 10:30 a.m. to 1:30 p.m.

Subject:PS01CSTA04:Statistical Inference I

Note: Figures to the right of questions indicate maximum marks. **Total Marks 70**

1. Choose the correct answer(s) and write in your answer book. **8**

(i) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n on X following geometric distribution with probability function $f(x,p) = pq^x$, $x = 0,1,2,\ldots$. Then which of the following are unbiased estimators of q/p.

(a) $T=\sum_{i=1}^{3} X_i /3$ (b) $T=X_1$ (c) $T=\sum_{i=1}^{n} X_i /n$ (d) all (a) to (c)

(ii) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n on X following B(1,p). Which of the following are true

(a) $\sum_{i=1}^{n} X_i(1-X_i^k)$ is unbiased for 0

(b) $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n} X_i^k$ are sufficient for p.

(c) $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n} X_i^k$ have the same distribution.

(d) All (a) through (c) are true.

(iii) Which of the following distribution does not belong to exponential family?

(a) Exponential distribution with mean θ

(b) Binomial distribution with mean 2p.

(c) Cauchy distribution with median θ.

(d) None of the above.

(iv) Let T be a complete sufficient statistics for the parameter θ.

(a) Then g(T) is sufficient for g(θ) but not complete.

(b) g(T) is both complete and sufficient for g(θ).

(c) g(T) is complete but not sufficient for g(θ).

(d) None of the above.

(v) The Fisher Information is equivalent to which of the following.

(a) It is the C-R lower bound for the variance of an estimator.

(b) It is a measure of information contained in the sample

(c) It is the variance of the score function.

(d) Both (b) and (c)

(vi) Let the distribution of X belongs to exponential family of distributions. Let T be a sufficient statistic. Then the statistics T also satisfies following condition.

(a) Statistics T is complete but does not have exponential family of distributions.

(b) Statistics T is complete and its distribution belongs to exponential family.

(c) Distribution of T belongs to exponential family but the statistic is not complete.

(d) None of the above.

(vii) Sample information and prior information leads to

(a) Posterior information

(b) Complete information

(c) Total information

(d) None of (a) to (c)

(viii) The CAN estimator of $g(\mu) = \mu^k - \mu$, (where μ is mean of the distribution) based on a random sample size n is

(a) $\overline{X}^k - \overline{X}$

(b) $\frac{\sum_{i=1}^{n} X_i^k}{n} - \overline{X}$

(c) Both (a) and (b)

(d) None of the above.

2. Answer any 7(**seven**) of the following.(Short Answer type) 2x7=14

(a) Obtain sufficient statistics for the parameters θ of the probability function

$$f(x;\theta_1,\theta_2) = \begin{cases} \frac{1}{\theta - 4}, & \text{if } 4 \le x \le \theta \\ 0 & \text{Otherwise} \end{cases}$$

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n on X following $N(0,\sigma^2)$ distribution. Then obtain Fisher information contained in the statistics $\frac{\sum_{i=1}^{n} X_i^2}{n}$ about σ^2.

(c) Show that family of normal distribution $N(\mu, 1)$, $-\infty<\mu<\infty$, is complete.

(d) Define score function and show that it is unbiased for 0. Also obtain its variance and comment on it.

(e) Define maximum likelihood estimator (m.l.e). Obtain maximum likelihood estimator of the parameter of the following distribution.

$$f(x,\mu) = \begin{cases} e^{-(x-\mu)} & \text{if } x > \mu \\ 0 & \text{otherwise} \end{cases}$$

(f) Give similarity and difference between CAN and BAN estimators.

(g) Show that the family of distribution given below belongs to two-parameter exponential family.

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, 0 < x < 1, \alpha, \beta > 0$$

(h) Obtain moment estimator of θ^k, where $k>0$ is an integer, based on a random sample of size n from exponential distribution with mean θ.

(i) Obtain posterior distribution of p when X follows geometric distribution with mean q/p and the prior distribution of p is β(r,s).

3. (a) State Neyman –Fisher factorization theorem and prove it when observations are drawn from one parameter discrete distribution. 6

(b) (i) Suppose $X_1, X_2, \ldots, X_n$ is random sample on X with probability density function $f(x;\theta,\gamma) = \frac{1}{\theta^{\gamma}\Gamma(\gamma)} x^{\gamma-1} e^{-\frac{x}{\theta}}$, $x>0$. Obtain sufficient statistic using factorization theorem. 3+3

(ii) Define minimal sufficiency. Show that the sufficient statistics obtained in (i) above are minimal sufficient.

OR

(b) (i) Define likelihood equivalence of two data point $\underline{x}=(x_1,x_2,\ldots,x_n)$ and $\underline{y}=(y_1,y_2,\ldots,y_n)$. With the help of an example show that likelihood equivalence 6 defines a statistics.

(ii) Show that set of order statistics is always sufficient.

4. (a) State and prove Lehman-Scheffe theorem. 6

(b) Show that Fisher Information contained in a statistics is always less than or equal to that contained in the complete sample. Show that when Fisher information contained in the statistics is equal to the information contained complete sample then the statistic must be sufficient statistic. 6

OR

(b) Define Fisher information matrix clearly giving all regularity conditions needed. Obtain the same when X follows $N(\theta_1, \theta_2)$ 6

5. (a) State and prove Cramer-Rao lower bound for the variance by clearly stating the necessary regularity conditions. 5

(b) (i) Let X be random variable with pdf $f_X(x,\theta)$ and $\theta=\varphi(\mu)$. Then Show $I_X(\mu)=\frac{I_X(\theta)}{\left(\frac{\partial\varphi(\mu)}{\partial\mu}\right)^2}$ 7

(ii) Let X be an exponential random variable with mean θ. Using Rao-Blackwellization obtain unbiased estimator for θ^2 as a function of sufficient statistics based on a random sample of size n.

OR

(b) Describe Roy-Mitra Technique to obtain unbiased estimators of parametric functions $(h(\theta))^r(g(\theta))^s$ when the probability function of X is of the form $f(x, \theta)=\frac{a(x)(h(\theta))^x}{g(\theta)}$, x>0. Use the result and obtain the UMVUE of density function of exponential distribution with mean θ. 7

6. (a) Obtain general expression for the posterior distribution of the parameter θ in the probability function f(x,θ). Show that Bayes estimator of θ under squared error loss function is the mean of the posterior distribution. 6

(b) Obtain the Bayes estimator θ and probability mass function when X follows B(1,θ) and the prior distribution of θ is beta with parameters α and β. 6

OR

(b) Write a note on CAN and BAN estimators. Also show that $\bar{X}^k$ is CAN estimator of θ^k when X follows B(1,θ) and $\bar{X}$ is based on a random sample of size n and k>0 is a positive integer. 6

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