OC

[921]

SARDAR PATEL UNIVERSITY
M.Sc External Examination, Semester -I
Saturday Date : 1-12- 2012
Time :10.30 a.m to 1.30 p.m
Subject/Course Code :PSO1CSTA02
Matrix Algebra

Q-1 Attempt following 08

(1) $\lambda = 3x_1 + 5x_2 + 7x_3$ then $\frac{d\lambda}{d\underline{x}} =$

(a) $\underline{0}$
(b) $(3, 5, 7)'$
(c) $(3, 5, 7)$
(d) Non of above.

(2) In usual notation : $A = [\underline{a}_1 \ \ \underline{a}_2] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow \underline{a}_1, \underline{a}_2$ are

(a) Linearly dependent.
(b) Orthogonal.
(c) Not linearly dependent but orthogonal.
(d) Not orthogonal and not linearly dependent.

(3) In usual notation for centering matrix C, $\underline{1}'C\underline{1}$

(a) Is zero.
(b) Is less than zero.
(c) Is greater than zero.
(d) Non of above.

(4) In usual notation Z_A denotes

(a) # of positive latent roots.
(b) # of negative latent roots.
(c) # of non-zero latent roots.
(d) # of zero latent roots.

(5) Signature of quadratic form is

(a) # of positive diagonal element.
(b) # of negative diagonal element.
(c) # of non-zero diagonal element.
(d) # of zero diagonal element.

(6) Vectors in basis of vector space

(a) Are unique.
(b) Are linearly dependent.
(c) Are not orthogonal.
(d) Are not unique.

(7) $A\underline{X} = \underline{0}$ has non-null solution vector if

(a) Matrix A is symmetric.
(b) Matrix A is singular.
(c) Matrix A is non-singular.
(d) Non of above.

(8) If $r = r_A$; rank of matrix A_{mxn} then

(a) $r \leq m.$
(b) $r \leq n$
(c) $r \leq \min \ (m, n)$
(d) $r \leq \max \ (m, n)$

PTO

Q-2 Attempt any **SEVEN** 14

(1) In usual notation explain unite length vector.
(2) Show that basis of vector space always exist.
(3) A^- is g-inverse of matrix A, if $AA^- A = A$. Give an another definition of A^-.
(4) Show that quadratic form is in variant under non-singular transformation.
(5) Derive latent roots of $A = A^3$.
(6) Define linearly independent vectors.
(7) Write method of obtaining latent vector.
(8) Write general solution of $A\underline{X} = \underline{b},\ \underline{b} \neq \underline{0}$
(9) In usual notation show that $X = XH$.

Q-3 A. Let A_i; $i = 1, 2, \ldots, k$ be square matrices of order m and $A = \sum_{i=1}^{k} A_i$. Consider following 06

(a) $A_i^2 = A_i$ for all i.
(b) $A_i A_j = 0$ for all $i \neq j, r_{A_i^2} = r_{A_i}$
(c) $A^2 = A$
(d) $r_A = \sum_{i=1}^{k} r_{A_i}$

Show that a, c $\Rightarrow$ b and a, b, c $\Rightarrow$ d.

B. In usual notation for inconsistent system of equations, $A\underline{X} = \underline{b}, \forall\, \underline{X} \in \Im$, show that $\|e(\underline{X})\| \geq \underline{b}'(H - I)\underline{b}$ where $H = A^- A$ and $e(\underline{X}) = (A\underline{X} - \underline{b})$. 06

OR

B. Explain data matrix, Uncorrected and corrected sum of squares of observation on j^{th} variable, corrected sum of product of observation on i^{th} and j^{th} variable. 06

Q-4 A. State and prove necessary and sufficient condition for matrix to be positive definite. 06

B. Explain : Canonical form under similarity and spectral decomposition of symmetric matrix. 06

OR

B. r^{th} moment of quadratic form $\underline{X}'A\underline{X}$; $\underline{X} \sim N(\underline{0}, \Sigma)$ is given by $\sum_{r=1}^{\infty} \left(\frac{t^r}{r!}\right) K_r = -\frac{1}{2} \log \|I - 2tA\Sigma\|$, then show that $K_r = 2^{r-1}(r-1)!\, \lambda_i(A\Sigma)^r$ and discuss special cases for $r = 1, 2$. 06

Q-5 A. Let $\Im$ be the linear manifold or subspace of solution vectors of homogeneous equations, $A\underline{X} = \underline{0}$ and $\mu = \mu(A)$ linear manifold of set A. Then show that $dim(\Im) = m - k$, where $dim(\mu(A)) = r_A = k$. 06

B. Derive latent roots of centering matrix. 06

OR

B. Show that g-inverse of any matrix always exist but not unique. 06

Q-6 A. State and prove Full-rank factorization. 06

B. Let $\Im$ be finite dimentional subspace and vector $\underline{\alpha} \notin \Im$. Then show that there exist vectors $\underline{\gamma}$ and $\underline{\beta} \ni \underline{\gamma} \neq \underline{0}$ is orthogonal to $\Im$ denoted by $\left(\underline{\gamma} \perp^{er} \Im\right)$ and $\underline{\alpha} = \left(\underline{\gamma} + \underline{\beta}\right)$; $\underline{\beta} \in \Im$. Further $\underline{\gamma}$ and $\underline{\beta}$ are unique. 06

OR

B. In usual notations prove following. (1) $r_{AB} \leq r_A$ *and* r_B
(2) Multiplying matrix by non-singular matrix does not alter rank. 06

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