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SARDAR PATEL UNIVERSITY
M.Sc. (I Semester) Examination
2012
Thursday, 29th November
10:30 a.m. to 1:30 p.m.
STATISTICS COURSE No. PS01CSTA01
(Probability Theory)

Note: Figures to the right indicate full marks of the questions. **(Total Marks: 70)**

1 Attempt all, write correct answers **08**

(i) For an infinte interval, _____ measure should be used because both the ________ ____ measure and the ___________ measure values are equal to ___
- a) L-S, Lebesgue, Counting, ∞
- b) L-S Measure, Lebesgue, Counting, 0
- c) Lebesgue, L-S, Counting, ∞
- d) Lebesgue, Counting, L-S, different

(ii) *Lim ∫ I(An) dP over Borel set B is_____ following _____*
- a) *P(A), MCT*
- b) *P(A∩B), DCT*
- c) P(A∩B), MCT
- d) P(A∩B), Fatous' theorem

(iii) Lebesgue measure is a particular case of L-S measure for choice of Function
- a) F(x) = x-2, x in R
- b) F(x) = x, x in R+
- c) F(x) = x, x in N
- d) None of the above

(iv) If $A_k = [1-1/k, 1+1/k)$ then $\cup A_k$ for $k \geq n$ is
- a) null set
- b) (1, 1)
- c)[1-1/n, 1+1/n)
- d)[1, 1)

(v) The field containing {1,2 ..., 100} has ___ elements
- a) 1
- b) cardinality of power set
- c) 100
- d) 2 raise to 100

(vi) The sufficient condition for a sequence of independent random variables having zero odd moments to hold strong law of large number is
- a) Variance is finite
- b) Mean is zero
- c) Third moment is finite
- d) Fourth moment is finite
- e)

(vii) The $< X_n >$ converges in probability to X then this imply
- a) $< X_n >$ converges in r^{th} mean
- b) $< X_n >$ converges a.s.
- c) $< X_n >$ converges in distribution
- d) None of these

(viii) The sequence of standardized sum of iid Bernouli random variables converges to a

a) Binomial r.v. b) Normal r.v.

c) Degenerate r.v. d) Poisson r.v.

2 **Attempt ANY 7, each carries 2 marks** **14**

(a) Prove that: If $A_n \to A$ then $A_n^c \to A^c$.

(b) Answer, what is $\lambda(0, 1/n]$= and $\lambda(N)$= and why?

(c) Show that indicator function of set A is measurable if and only if A is a measurable set.

(d) Prove in usual notations that $\int_\Omega s + t \, d\mu = \int_\Omega s \, d\mu + \int_\Omega t \, d\mu$

(e) Using Jenson's inequality prove that $E^{1/r}|X|^r \leq E^{1/s}|X|^s$ for $0<r<s$.

(f) State and prove Borel-Cantelli lemma.

(g) Let X_n be a sequence of random variables defined by $P(X_n = 0) = 1-1/n^r$ and $P(X_n = n) = 1/n^r$ $r>0$, $n \geq 1$. Verify that $X_n \to 0$ in probability but not in rth mean.

(h) Show that $F(x) = \begin{cases} 0 & x < 0 \\ x/2 & 0 \leq x \leq 1 \\ 1 - x + x^2/2 & 1 \leq x \leq 2 \\ 1 & 2 \leq x \end{cases}$ is continuous distribution function.

(i) Let $\{X_n\}$ be sequence of independent random variables with common uniform distribution over (0, 1). For $A_n = (X \leq 1/n)$, find the probability $P(\limsup A_n)$.

(j) If $X_1, X_2, \ldots, X_n$ are iid bernouli random variables find the characteristic function of $Y = \sum_1^n X_i$.

3(a) Define semi-field, field and sigma field giving example considering $\Omega = (0,1]$. What are the interrelationships among these classes? **06**

3(b) Define counting measure, Lebesgue measure and Lebesgue-Steiltjes measure. Show that each measure function is sigma finite and can be extended to a probability measure. **06**

OR

3(b) Show that probability measure is continuous.

4(a) Define Borel measurable function. Show that if f is non-negative measurable function then $f^2 + \alpha$ is also non-negative measurable, α is a real constant. **06**

4(b) Show that increasing sequence of non-negative simple functions converges to a non-negative measurable function. **06**

OR

4(b) State and prove monotone convergence theorem.

5(a) Prove that, if $y=g(x)$ is differentiable for all x, and either $g'(x) > 0$ or <0 for all x and if X is continuous then $Y=g(X)$ is continuous. Also obtain the density function of Y. If the pdf of random variable X is $f(x) = \alpha\beta x^{\beta-1} \exp(-\alpha x^\beta)$ for $x \geq 0$ and 0 otherwise, then find pdf of $y = x^\beta$. **06**

5(b) State and prove Basic inequality. **06**

OR

5(b) If X is a random variable taking values –t, 0, t with probabilities p, 1-2p, p respectively, show that Markov's inequality attains equality.

6(a) Show that characteristic function is uniformly continuous and differentiable twice if first and second moments are finite. **06**

6(b) State and prove Kintchin's weak law of large numbers. Using this what can you say about the sequence <X_n> having pmf $P(X_n = n) = \frac{1}{2n} = P(X_n = -n)$ and $P(X_n = 0) = 1 - \frac{1}{n}$. Does this sequence hold Weak law any way? **06**

OR

6(b) State and prove Lindberg-Levy central limit theorem (CLT). State one application of this CLT.