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SEAT No.________

[96]

SARDAR PATEL UNIVERSITY
M.Sc. (I Semester) Examination
2019
Monday, 25th March
10:00 a.m. to 1:00 p.m.
STATISTICS COURSE No. PS01CSTA23
(Distribution Theory)

Notes: Figures to the right indicate marks. (Total marks: 70)

1 Wright the correct answer (each question carries one mark). **8**

(a) Whether a distribution is central or non-central depends on
(A) location parameter of basic distribution (B) scale parameter of basic distribution
(C) mean of basic distribution (D) none.

(b) A distribution with truncated range is known as
(A) regular distribution (B) non-regular distribution
(C) mixture distribution (D) truncated distribution

(c) Area between two ordered observations under a density function is
(A) one (B) symmetric about mean
(C) distribution free statistic (D) none

(d) A test statistic for the sign test is
(A) order statistic (B) rank order statistic
(C) distribution free statistic (D) all of the above

(e) A correlation between ________________ is called partial correlation.
(A) two variables (B) a variable and a set of variables
(C) two residuals (D) between two sets of variables

(f) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ then $\underline{X}'A\underline{X}$ and $\underline{X}'B\underline{X}$ are independent iff
(A) $A\Sigma B = 0$ (B) $B\Sigma A = 0$
(C) $AB = BA = 0$ (D) None of these

(g) A test statistic used in sign-test follows
(A) Bernoulli distribution (B) Poisson distribution
(C) Binomial distribution (D) Normal distribution

(h) Rank order statistics are
(A) normally distributed (B) uniformly distributed
(C) exponentially distributed (D) none of the above

2 Answer any SEVEN of the following (each question carries two marks) **14**

(a) Suppose X and Y are independent Poisson variables with parameters λ and μ. Show that (i) X + Y is also Poisson, (ii) the condition distribution of X given X + Y is binomial.

(b) Let X follows binomial distribution with mean np. It is given that X never assumes the values 0 then fine E(X) under this condition.

(c) Obtain first two moments of a non-central chi-square distribution.

(d) Define sample median and obtain its density.

(e) If $Y_1 < Y_2 < Y_3$ are the order statistics of a r.s from a distribution with density

(P.T.O.)

$f(x) = 1$ if $\theta - \frac{1}{2} < x < \theta + \frac{1}{2}$. Show that $P[\theta - 0.4 < Y_2 < \theta + 0.4] = 0.944$.

(f) What is regression? Show that it is conditional mean.

(g) Let $\underline{X}$ be distributed $N_3(\underline{\mu}, \Sigma)$ with the pdf $f(\underline{x}) = Const \cdot \exp\{-Q/2\}$ where

$Q = 3X_1^2/2 + 2X_2^2 + X_3^2 - 3X_1X_2 - 2X_1X_3 - 2X_2X_3 + 10X_1 - 14X_2 + 8X_3 + 26$

Find k.

(h) Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ where $\underline{\mu}' = (2,1,1)$ and $\Sigma = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. Obtain the joint distribution of $Y_1 = X_1 + X_2 + X_3$ and $Y_2 = X_1 - X_2$.

(i) For the distribution given in 2(h), find $\rho_{12\cdot3}$ and $\rho_{1\cdot23}$.

3 (a) Let $X_1, \dots, X_n$ be iid N(0, 1) variables. Obtain the distribution of $Y = X_1 / \sqrt{\sum_{i=1}^{n} X_i^2 / n}$ 6

(b) Define non central chi-square statistic with one d.f. and derive its pdf. 6

OR

(b) Define non-central F variable and derive its density and first moment.

4 (a) Define partial correlation coefficients. Derive its expressions in terms of elements of Σ and Σ^{-1}. 6

(b) Define nonsingular and singular multinomial distribution. Prove that the marginal distribution of a subset $X_{r+1}, \dots, X_k$ of $X_1, \dots, X_k$ having multinomial distribution is nonsingular. 6

OR

(b) Discuss: Transformation of statistics and its roll.

5 (a) Write a note on rank ordered statistics. 6

(b) Show that the area under the density function between any two ordered observations is distribution free statistic. Find the distribution of the area of f(x) between $X_{(1)}$ and $X_{(n)}$. 6

OR

(b) Write note on extreme values and their asymptotic distributions.

6 (a) Define multivariate normal distribution. State its properties. Prove any one of them. 6

(b) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$, $\Sigma > 0$. Prove that the distribution of $\underline{X}' A \underline{X}$ is non-central chi-square if and only if $A\Sigma$ is idempotent matrix. 6

OR

(b) Let $\underline{X} \sim N_3(\underline{\mu}, I)$ where $\underline{\mu} = (3, -2, 1)'$. Also, let

$A = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$, $B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ and $C = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Answer any two of the following:

(i) Obtain the distribution of $\underline{X}'A\underline{X}$?

(ii) Are $\underline{X}'A\underline{X}$ and $B\underline{X}$ independent? Verify.

(iii) Are $\underline{X}'A\underline{X}$ and $\underline{X}'C\underline{X}$ independent? Verify.

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