(32) **SEAT No.________**

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SARDAR PATEL UNIVERSITY
M.Sc. (I Semester) Examination
2019
Tuesday, 19th March
10:00 a.m. to 1:00 p.m.
STATISTICS COURSE No. PS01CSTA21 (Probability Theory)

Note: Figures to the right indicate full marks of the questions. **(Total Marks: 70)**

1 Attempt all, write correct answers **08**

(i) Every sigma field is also a

a) monotone class
b) class of subsets of infinite Ω
c) class defined on R
d) countable set

(ii) Limit set of an increasing sequence of sets $\{B_n, n \geq 1\}$ is

a) $\cap B_n$
b) $\cup B_n$
c) null set
d) Ω

(iii) The probability is additive provided

a) class of events is a Power set
b) class of events are pair wise disjoint
c) class is (Φ, Ω)
d) None of these

(iv) Every linear combination of indicator variable of measurable set is ....

a) a simple random variable
b) a non-negative random variable
c) a constant function
d) a random variable

(v) The singular distribution function is a _____ distribution function.

a) continuous
b) discrete
c) unbounded
d) mixed

(vi) A sufficient condition for a sequence of independent random variables having zero odd moments to hold strong law of large number is proved using ____________

a) Markov's inequality
b) Holder's inequality
c) Chebychev's inequality
d) Jenson's inequality

(vii) A sequence of random variables $< X_n >$ converges in rth mean to a random variable X then this imply

a) $< X_n >$ converges in $(r+1)^{th}$ mean
b) $< X_n >$ converges a.s.
c) $< X_n >$ converges in distribution
d) Convergence in probability

(viii) The sequence of standardized sum of iid Bernouli random variables converges to a

a) Binomial r.v.
b) Normal r.v.
c) Degenerate r.v.
d) Cauchy r.v.

2 Attempt ANY 7, each carries 2 marks **14**

(a) Let $\{A_n = [1-1/n, 1+1/n), n \geq 1\}$ from a sample space. Obtain limit set A.

(b) Define Lebesgue measure and Lebesgue-Steiltjes measure giving example.

(P.T.O.)

(c) Show that intersection of fields is also a field.

(d) Prove in usual notations that $s + t$ is a simple random variable given that, s and t are two simple random variables.

(e) State and prove Liaponov's inequality.

(f) State Borel 0-1 law and prove the first part.

(g) Check whether the following F is a distribution function and identify its type. If F is mixed then decompose it.

$$F(x) = \begin{cases} \frac{3}{4} + (1-e^{-\beta x})\big/4, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

(h) If $X_1, X_2 ..., X_n$ are iid normal random variables find the characteristic function of $Y = \sum_1^n X_i$.

(i) State Liaponov's CLT. How it is different from the Lindberg-Levy CLT?

3(a) Define field and sigma field giving example considering Ω = real line. Are these field and sigma field the Borel field and Borel sigma field? **06**

3(b) Stating five properties of probability measure, prove one of these. **06**

OR

State and prove Basic inequality. State which inequalities are derivable from it.

4(a) Show that if X and Y are two is non-negative random variable then X+Y is also non-negative random variable. Using this show that, $X + \alpha$ is also non-negative random variable, α is a real constant. **06**

4(b) Show that increasing sequence of non-negative simple random variables converges to a non-negative random variable. **06**

OR

State and prove Jordan decomposition theorem. Define characteristic function of a mixture distribution function.

5(a) State and prove monotone convergence theorem. Also state how MCT is used in proving Fatou's theorem. **06**

5(b) Define in usual notation EX, where X is a random variable with respect to a Lebesgue measure. Hence find EX, given, Ω = (-5, 5) and X on (Ω, A, λ) takes values $X(\omega) = 1/2$, $\omega \in (-5,2)$, $X(\omega) = 1/3$, $\omega \in [2,2]$, $X(\omega) = 1$, $\omega \in (2,3]$ and $X(\omega) = 0$, $\omega \in (3,5)$. **06**

OR

Define and discuss the interrelationship among the modes of convergences of a sequence of random variables.

6(a) Show that characteristic function is uniformly continuous and differentiable if first moment is finite. **06**

6(b) State and prove weak law of large numbers for iid random variables. Apply the same to <X_n> having pmf $P(X_n = n) = \frac{1}{2n} = P(X_n = -n)$ and $P(X_n = 0) = 1 - \frac{1}{n}$. Does this sequence hold Weak law any way? **06**

OR

State and prove Lindberg-Levy central limit theorem (CLT). State one application of this CLT.

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