SEAT No.________

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SARDAR PATEL UNIVERSITY

M.Sc.(Statistics) First Semester Examinations
Wednesday, March 27, 2019
Time 10:00 a.m. to 1:00 p.m.
Subject:PS01CSTA04/PS01CSTA24:Statistical Inference I

Note: (i) Figures to the right of questions indicate maximum marks.
(ii) Total Marks is 70

1 Choose the most correct answer and write in your answer book. 8

(i) Y= $aX_1 + (1-a)X_2$ where X_1 and X_2 are independent and follow N(θ,1) and double exponential with mean θ respectively. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample on Y. Then the unbiased estimator of θ based on the random sample is

(a) $T_1 = \frac{Y_1 + \sum_{j=2}^{n}(jY_j - (j-1)Y_{j-1})}{n}$ (b) $T_2 = \sum_{i=1}^{n} b_i Y_i$ where $\sum_{i=1}^{n} b_i = 1$

(c) $T_3 = \bar{Y}$ (d) All (a) to (c)

(ii) Let X_1, X_2 and X_3 are independent random variables with X_i following $B(n_i, p)$, i=1,2,3. Which of the following is true?

(a) $\frac{X_1}{6n_1} + \frac{2X_2}{6n_2} + \frac{3X_3}{6n_3}$ and $\frac{X_1+X_2+X_3}{n_1+n_2+n_3}$ both are unbiased for p.
(b) $\frac{X_1}{3n_1} + \frac{X_2}{3n_2} + \frac{X_3}{3n_3}$ is sufficient for p.
(c) Only the given data is sufficient.
(d) None of the above.

(iii) Let X be a geometric random variable with density $f(x,\theta) = \theta(1-\theta)^{x-1}$, x =1,2,… . and $X_1, X_2, \ldots, X_n$ be a random sample on X. Then which of the following is true?

(a) $\sum_{i=1}^{n} X_i$ is sufficient for θ. (b) $\sum_{i=1}^{n}(X_i - 1)$ is sufficient for θ.
(c) $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n}(X_i - 1)$ are minimal sufficient (d) None of the above

(iv) Let $X_1, X_2, \ldots, X_n$ be random sample on X which follows Poisson distribution with mean θ. Define $n\,T_1 = \sum_{i=1}^{n} X_i$ and $(n-1)T_2 = \sum_{i=1}^{n}(X_i - \bar{X})^2$. Then which of the following is most correct?

(a) Both T_1 and T_2 are UMVUE for θ.
(b) Both T_1 and T_2 are unbiased for θ with T_1 being UMVUE.
(c) Both T_1 and T_2 are unbiased for θ
(d) Both T_1 and T_2 are sufficient for θ

(v) Which of the following does not satisfy C-R regularity conditions?

(a) Binomial distribution
(b) Exponential distribution with support (μ,∞).
(c) Negative binomial distribution
(d) Geometric distribution.

(vi) Which of the following distribution does not belong to exponential family?

(a) Truncated exponential distribution $f(x,\theta) = e^{-(x-\theta)}, x > \theta$.
(c) Truncated geometric distribution with $f(x,\theta) = \theta(1-\theta)^{x-5},\ x = 5,6,\ldots$

(P.T.O)

(b) Binomial distribution with mean 2p (d) None of (a) to (c).

(vii) Consider a type II censored sample of size r from a life experiment with n identical units working independently. Further assume that life times follow exponential distribution with mean θ. Then which one of the following is correct?

(a) The total time of test is sufficient for θ (b) The total time on test is UMVUE of θ

(c) Both (a) and (b) (d) None of (a) to (c)

(viii) Which one is most correct?

(a) Fisher Information contained in the statistics is always less than or equal the Fisher Information contained in the given sample.

(b) Fisher Information does not exist for $f(x,\theta) = 0\ if\ x \le \theta$
$= e^{-(x-\theta)} if\ x > \theta$

(c) We cannot compute Fisher Information if range depends on the parameter θ.

(d) All (a) to (c)

2 Answer any **seven** of the following:(Short Answer type) 14

(a) Define sufficient statistics. Consider a random sample X_1, X_2 from B(1,θ) . Let T= $X_1 + X_2$. Obtain conditional distribution of (X_1, X_2) given T. Comment on the nature of statistic T based on the distribution.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from N(μ_0, σ^2). Then obtain Fisher information contained in the sufficient statistics about σ^2.

(c) Let X follows exponential distribution with mean θ. Obtain unbiased estimator of P(X>kθ), k>0, based on a random sample of size n.

(d) Define completeness of a statistics T. Show that family of normal distribution N (μ, 1), $-\infty<\mu<\infty$, is complete.

(e) Consider two parameter exponential distribution with density function

$$f(x;\mu,\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{(x-\mu)}{\theta}} & if\ x > \mu \\ 0 & Otherwise \end{cases}$$

Obtain sufficient statistics for (μ, θ) based on random sample size n from the distribution.

(f) Show that score function has the variance equal to Fisher Information contained in the data about the parameter θ in the distribution f(x,θ).

(g) Show that moment estimator of mean of geometric distribution is asymptotically unbiased.

(h) Obtain posterior distribution of the mean λ of Poisson distribution when the prior distribution of λ is exponential with mean 1.

(i) Obtain maximum likelihood estimator of θ based on n i.i.d observations following the density f(x, θ)= exp(-(x- θ)) , x > θ.

3 (a) Define Fisher Information contained in the random variable X about the parameter θ in its probability function f(x;θ). Obtain the relation between Fisher Information contained in a statistics and in the random sample. 6

(b) State Neyman-Fisher factorization theorem. Prove the theorem when X is discrete. 6

OR

(b) Define minimal sufficient statistic. Let $\underline{x} = (x_1, x_2, \ldots, x_n)$ and $\underline{y} = (y, y_2, \ldots, y_n)$ be any two 6 data points from the samples space of $\underline{X}$, a random sample of size n on X. Let $\frac{f_{\underline{X}}(x,\theta)}{f_{\underline{X}}(y,\theta)} = h(\underline{x}, \underline{y}, \theta)$. Then prove that $T(\underline{X})$ is minimal sufficient for θ provided $h(\underline{x}, \underline{y}, \theta)$ is independent of θ if and only if $T(\underline{x}) = T(\underline{y})$.

4 (a) Giving the necessary regularity conditions state and prove Cramer-Rao Lower bound for the variance of the unbiased estimator of ψ(θ) when X has the probability function f(x,θ). 6

(b) Write a detailed note on Exponential family of distributions and their uses. 6

OR

(b) (i) Let $X_1, X_2, \dots, X_n$ be random sample from 3+3

$$f(x,\theta) = \begin{cases} \frac{1}{(b(\theta) - a(\theta))} & if\ a(\theta) < x < b(\theta) \\ 0 & Otherwise \end{cases}$$

where a(θ) is increasing and b(θ) is decreasing function of θ. Then obtain sufficient statistics based on a random sample of size n on X.

(ii) Give an example of sufficient statistic which is not complete.

5 (a) State separately Rao-Blackwell and Lehman Scheffe theorems. Prove Rao-Blackwell theorem. 6

(b) Consider following family of distributions. 6

$$f(x,\theta) = \begin{cases} \frac{a(x)(h(\theta))^{d(x)}}{g(\theta)} & if\ x \in \varkappa \\ 0 & otherwise \end{cases}$$

Where a(x), $h(\theta)$ and $g(\theta)$ are analytic functions of corresponding arguments.

Using Roy-Mitra Technique obtain UMVUE of $(\theta) = (h(\theta))^r (g(\theta))^s$, r, s > 0.

OR

(b) State and Prove Battacharyya system of lower bounds in the one parameter case. 6

6 (a) Obtain the Bayes estimator of the parameter θ in one parameter family of distributions f(x,θ), under the squared error loss function. 6

Suppose X follows B(n,θ). Consider the Jeffereys prior $\pi(\theta) \propto \sqrt{I(\theta)} = \sqrt{\frac{n}{\theta(1-\theta)}}$. Under this prior obtain Bayes estimator of θ using squared error loss function.

(b) Show that moment estimator, based on random sample of size n, of the parametric function $\psi(\theta) = (1-\theta)^r \theta^s$ is consistent; here $r, s \geq 1$, and θ is the mean of exponential distribution. Obtain moment estimator, based on random sample of size n, of the parameter θ from U(θ, b) where b is a constant and θ < b . Is it same as mle of θ?. 6

OR

(b) Define CAN and BAN estimators. Let X follows Poisson distribution with parameter λ. Then show that maximum likelihood estimator of λ based on random sample of size n is CAN estimator 6

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