(104 & A-34) Seat No.:________

No. of Printed Pages : 3

SARDAR PATEL UNIVERSITY

M. Sc. (Physics) 3rd Semester Examination
Friday, 21st October, 2016
Time: 02:00 pm to 05:00 pm
Subject: PS03CPHY01 [Quantum Mechanics-II]

Total Marks: 70

Note: (1) Figures to the right indicate marks.
(2) Symbols have their traditional meaning.

Q:1 Attempt all of the following Multiple choice type questions. [01 mark each] [08]

(1) For Pauli matrices $\sigma_+^2 =$ ______. Take $\hbar = 1$.

(a) 0
(b) $-i\sigma_+$
(c) $i\sigma_-$
(d) -1

(2) In the matrix representation of angular momentum $j = \frac{1}{2}$, $J_+ =$ ______. Take h = 1.

(a) $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(d) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

(3) When the time variation of the Hamiltonian is small we take

(a) dipole approximation
(b) sudden approximation
(c) Born approximation
(d) adiabatic approximation

(4) $a_f^1(t)$ provides a good approximation to the $a_f(t)$ if the total probability for transitions from i to all states $f \neq i$ is

(a) $\sum \left|a_f^1(t)\right|^2 \ll 1$
(b) $\sum \left|a_f^1(t)\right|^2 \gg 1$
(c) $\sum \left|a_f^1(t)\right|^2 = 1$
(d) $\sum \left|a_f^1(t)\right|^2 = 0$

(5) $V(t,t_1)V(t_1,t_0) =$

(a) $V(t_0,t_1)$
(b) 1
(c) $V(t,t_0)$
(d) $V(t_0,t)$

(6) In the transformation $|\Psi\rangle \rightarrow U|\Psi\rangle$, choosing U as $V^{-1}(t,t_0)$ leads to

(a) Schrödinger picture
(b) Dirac picture
(c) Heisenberg picture
(d) Interaction picture

(7) In Natural units ($\hbar = c = 1$), dimensional formula for electric charge is ______.

(a) $M^1L^0T^0$
(b) $M^0L^0T^0$
(c) $M^1L^0T^{-1}$
(d) $M^1L^1T^0$

(P.T.O.)

(8) The Klein-Gordon field is an example of spin s = ______ field.

(a) 0
(b) 1
(c) 5/2
(d) 3/2

Q:2 **Answer any 7 of the following 9 questions briefly. [02 marks each]** [14]

1. For $\vec{J} = \vec{J}_1 - \vec{J}_2$, prove that $\vec{J}_1 - \vec{J}_2$ does not represent angular momentum.
2. Define *Zeeman* effect. Give difference between anomalous and normal Zeeman effect.
3. Draw the curve of transition probability as a function of energy difference between initial and final states, for fixed t in case of constant perturbation.
4. What is Dipole approximation?
5. Define and differentiate between elastic and inelastic scattering.
6. Explain briefly the Schrödinger picture.
7. (i) Why is it necessary to consider Dirac's relativistic Hamiltonian?
   (ii) Write down Pauli matrices.
8. For S.H.O., write lowering ($\hat{a}$) and ($\hat{a}^{+}$) operators in terms of position and linear momentum operators. Prove that $\left[\hat{a}^{+}, \hat{a}\right] = -1$.
9. Define field. Write its coordinate.

Q:3 (a) Derive total non-relativistic Hamiltonian for atomic system including spin in presence of external magnetic field. Write and interpret energy Eigen values expression for a case when spin-orbit interaction is neglected. [6]

(b) Derive eigenvalue spectrum for j_z and J^2, when $[J^2, j_z] = 0$. [6]

OR

(b) For $\left\langle j'm' \middle| J_+ \middle| jm \right\rangle = C^{+}_{jm}\hbar\delta_{j'j}\delta_{m',m+1}$, obtain an expression for C^{+}_{jm}. [6]

For two independent (non-interacting) angular momentum vectors $\vec{J}_1$ and $\vec{J}_2$ deduce addition of them, and show that it is equivalent to old '*vector model*'. Define Clebsch-Gordan coefficients.

Q:4 (a) Obtain the general solution of time-dependent Schrödinger equation. [6]

(b) Discuss in detail the interaction of an atom with electromagnetic radiation. [6]

OR

(b) Derive Fermi's golden rule and give its interpretation. [6]

Q:5 (a) Derive the Klein Gordon equation. Show that the plane wave solution of the equation gives $E = \pm(c^2\vec{p}^2 + m^2c^4)^{1/2}$. [6]

(b) Explain the Heisenberg picture of time evolution. [6]

OR

(b) Starting with Dirac's relativistic Hamiltonian $H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$ derive the Dirac equation. [6]

Q:6 (a) Derive Lagrangian classical field equation. Using the concept of functional derivative, obtain classical field equation analogous to Langrange's equation for a system of particles. [6]

(b) Derive Hamiltonian form of field equation. For a dynamical physical quantity F as a functional of ψ and $\prod$, obtain its time rate of change and introduce the definition of Poisson bracket for field coordinates. [6]

OR

(b) Explain the meaning of *second quantization.* Assuming following form for Lagrangian density, viz; $\mathcal{L} = i\hbar\psi^*\dot{\psi} - \frac{\hbar^2}{2m}(\nabla\psi^*)(\nabla\psi) - V(\vec{r},t)\psi\psi^*$, and using Hamiltonian form for field equation derive time dependent Schrödinger equation. [6]

— X —