SC

Total No. of Printed Pages: 02.

No. Of Printed Pages: 2

[394]

SARDAR PATEL UNIVERSITY
M. Sc. Physics IIIrd Semester Examination
Tuesday, Date: 05-04-2016 Time: 02.30 to 5.30 PM
CBCS Course No.: PS03CPHY01
Subject: Quantum Mechanics–II

Note: Symbols have their usual meaning. **Total Marks: 70**

Q.1 Write answers of all eight questions in a table form by showing your choice against the question number. **(8)**

(1) The *gyromagnetic* ratio associated with *spin* has the ______ value than the one associated to *orbital* motion.
(a) same (b) double (c) half (d) four-times

(2) Which of the following transition is electric dipole allowed?
(a) $1s \rightarrow 2s$ (b) $2p \rightarrow 3d$ (c) $3s \rightarrow 4f$ (d) $3s \rightarrow 5d$

(3) The coordinate representation of time evolution operator in *interaction* picture leads to ______.
(a) Dirac matrix mechanics (b) Schrödinger representation
(c) uncertainty principle (d) Feynman diagram

(4) For Pauli matrices; $[\sigma_x, \sigma_y]$ = ______. Take $\hbar = 1$.
(a) $-i\sigma_z$ (b) $-i\sigma_+$ (c) $i\sigma_+$ (d) $i\sigma_z$

(5) The spin-orbit interaction Hamiltonian is directly proportional to ______.
(a) $r\frac{dV}{dr}$ (b) $\frac{1}{r}\frac{dV}{dr}$ (c) $\frac{1}{r^2}\frac{dV}{dr}$ (d) $r^2\frac{dV}{dr}$

(6) In the matrix representation of angular momentum $j = ½$, J_+ = ______. Take $\hbar = 1$.
(a) $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

(7) In Natural units ($\hbar = c = 1$), dimensional formula for energy E is ______.
(a) $M^1L^0T^0$ (b) $M^1L^1T^0$ (c) $M^1L^{-1}T^0$ (d) $M^1L^0T^{-1}$

(8) For Dirac matrices α_x, α_y, α_z and β, the trace of α_x, i.e., tr α_x, is ______.
(a) - tr α_y (b) - tr α_x (c) - tr α_z (d) tr β

Q.2 Answer any seven questions. **(14)**

(1) Prove that $s_+\alpha = 0$, and also give reason for it.
(2) For two angular momenta $\vec{J_1}$ and $\vec{J_2}$, prove that the difference $\vec{J_1} - \vec{J_2}$ does not represent angular momentum.
(3) Using time dependent Schrödinger equation, deduce the integral form for propagator.
(4) Prove that $c\boldsymbol{\alpha}$ can be interpreted as the velocity operator. Here, $\boldsymbol{\alpha}$ stands for Dirac matrix.
(5) Give difference between Schrödinger picture and Heisenberg picture.
(6) Show that the angular momentum is not a constant of motion in Dirac theory.
(7) Write conditions for field co-ordinates and conjugate momentum to quantize the field.
(8) Define *field*. Explain its co-ordinates.

(9) What is Born approximation? When is it suitable for explaining scattering phenomena?

Q.3 **(a)** For $[S^2, s_z] = 0$, expand any spin state $|\chi\rangle$ in terms of complete orthonormal eigenstates $|s, m_s\rangle$. Obtain spin wave functions for $s = ½$. **(6)**

(b) Derive an expression for non-relativistic Hamiltonian including spin. Obtain an expression for corresponding energy eigenvalues. **(6)**

OR

(b) Explain the construction of the basis states for a system defined by addition of two angular momenta, $\vec{J_1}$ and $\vec{J_2}$, when (i) $[\vec{J_1}, \vec{J_2}] = 0$ and (ii) $[\vec{J_1}, \vec{J_2}] \neq 0$. What are CG coefficients? **(6)**

Q.4 **(a)** For the case of constant perturbation show that the transition probability is given by $\left|a_f^{(1)}(t)\right|^2 = \frac{\left|H'_{fi}\right|^2}{\hbar^2} \frac{4\sin^2(\frac{\omega_{fi}t}{2})}{\omega_{fi}^2}$, and explain it. **(6)**

(b) Explain scattering of a particle by potential, and derive an equation for differential scattering cross section. **(6)**

OR

(b) Explain the time-dependent perturbation technique. Discuss with schematic diagram the first and second order transitions. **(6)**

Q.5 **(a)** (i) Derive relativistic wave equation. **(3)**

(ii) Briefly explain Dirac relativistic Hamiltonian. **(3)**

(b) Write note on Schrodinger picture for time evolution of quantum system. **(6)**

OR

(b) Obtain the positive and negative energy solutions of a free Dirac particle and interpret these solutions. **(6)**

Q.6 **(a)** Derive classical field equation in terms of Lagrangian density ($\mathcal{L}$). Using the notion of functional derivative, deduce $\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\Psi}}\right) - \frac{\partial L}{\partial \Psi} = 0$. **(6)**

(b) Derive classical field equation in Hamiltonian form. **(6)**

OR

(b) Using an expansion $\Psi(\vec{r}; t) = \sum_k a_k(t) u_k(\vec{r})$, derive $N_k = a_k^\dagger a_k$. Prove that the eigenvalues of N_k are all positive integers. Define the *vacuum state*. Show that $[a_k, N_k] = a_k$. **(6)**

****