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## SARDAR PATEL UNIVERSITY

M. Sc. (Physics) (II<sup>nd</sup> Semester) Examination

Day: Monday, Date: 09/04/2018, Time: 02:00 p.m. to 05:00 p.m.

Course No. PS02CPHY21 (Classical and Quantum Mechanics)

Important Note: Q.1: Eight multiple choice questions (MCQ) carry one mark each.

Q.2: Short answer questions carry two marks each (attempt any seven out of nine).

Q.3 to Q.6: Long answer questions carry 12 marks each.

Total Marks: 70

- Q1 (i) The physical quantities in Lagrangian formulation are  
 (a) vector (b) scalar  
 (c) tensor (d) none of them
- (ii) The solution of  $\frac{\partial H}{\partial q_i}$  is \_\_\_\_\_;  
 (a)  $-p_i$  (b)  $-q_i$  (c)  $p_i$  (d)  $q_i$
- (iii) If  $P_k = P_k(p_i, t)$ , then which generating function will not be used \_\_\_\_\_.  
 (a)  $F_3(p, Q, t)$  (b)  $F_4(p, P, t)$  (c)  $F_1(q, Q, t)$  (d)  $F_2(q, P, T)$
- (iv) The equilibrium is said to be unstable if extremum value of potential energy  $V$  is a  
 \_\_\_\_\_.  
 (a) maximum (b) minimum  
 (c) zero (d) any value between minimum and maximum
- (v)  $\frac{1}{2}(J_+ + J_-) =$   
 (a)  $J_y$  (b)  $J_x$  (c) 0 (d)  $J_z$
- (vi) \_\_\_\_\_ plays the role of generator of infinitesimal translations.  
 (a)  $p_z/\hbar$  (b)  $L_z/\hbar$  (c)  $\hbar L_z$  (d)  $\hbar p_z$
- (vii)  $\sum_{a,i} |ai\rangle\langle ai| =$   
 (a)  $\delta_{ai}$  (b)  $\hat{1}$  (c)  $\delta_{ii}$  (d) 0
- (viii) The azimuthal quantum number  $m$  can assume the values \_\_\_\_\_  
 (a)  $-j, -j+1, \dots, 0, \dots, j-1, j$  (b)  $-j+1, \dots, 0, \dots, j-1$   
 (c)  $0, \dots, j+1$  (d)  $-j+1, \dots, 0$
- Q2 (i) Discuss symmetry properties of a system in motion and Noether's theorem.
- (ii) The Hamilton-Jacobi equations are first-order partial differential equation having  $(n+1)$  variables, discuss.
- (iii) Discuss in brief the Poisson's brackets. Prove that Poisson's bracket obeys distributive laws of algebra.
- (iv) Discuss the secular equation for small oscillations.
- (v) Discuss theory of small oscillations and explain with example its equilibrium states.

(vi) Show that any operator is diagonal in its own representation.

(vii) Explain product of operators.

(viii) Find  $[J_x, J^2]$ .

(ix) What are C-G coefficients?

**Q3** (a) Using D'Alembert's principle, derive the second order Lagrangian's differential equation of motion for a conservative holonomic system. 6

(b) Prove that Poisson brackets are invariant under canonical transformation. Obtain canonical equations in terms of Poisson bracket notation and discuss about its symmetry with respect to the corresponding differential equations. 6

OR

(b) What do you understand by gauge and canonical transformation? Obtain a relation between new and old coordinates, momentum and Hamiltonian function. What are generating functions? 6

**Q4** (a) Discuss the problem of one-dimensional harmonic oscillator having Hamiltonian  $H = p^2/2m + m\omega^2 q^2/2$ ,  $\omega = (k/m)^{1/2}$ . Derive the necessary equations using Hamilton-Jacobi method. Here symbols have usual meaning. 6

(b) Discuss the theory of the free vibrations of linear triatomic molecule with necessary mathematical equations. 6

OR

(b) Show that the eigenvectors corresponding to the two distinct eigen frequencies are orthogonal. Explain the meaning of orthogonality. 6

**Q5** (a) Explain representation of state vectors and deduce the relation  $(\chi)_A = [F]_A (\psi)_A$ . 6

(b) Explain the unitary transformation induced by rotation of coordinate system and show that  $|x\rangle' = e^{-i\theta_n \hat{L}_n / \hbar} |x\rangle$ . 6

OR

(b) Discuss the algebra of rotation generators. 6

**Q6** (a) Obtain the eigen value spectrum of  $J^2$  and  $J_z$ . 6

(b) Discuss the matrix representation of  $\mathbf{J}$  in the  $|jm\rangle$  basis. 6

OR

(b) Determine the spin wave function for a system of two spin half particles and discuss the triplet and singlet state. 6

