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[61]

SARDAR PATEL UNIVERSITY

M. Sc. Physics II[nd] Semester Examination

Monday, Date: 04-04-2016 Time: 10.30 AM to 01.30 PM

CBCS Course No.: PS02CPHY01

Subject: Quantum Mechanics–I

Note: Symbols have their usual meaning. **Total Marks: 70**

Q.1 Write answers of all eight questions in a table form by showing your choice against the question number. **(8)**

(1) First excited state of hydrogen atom is _____ fold degenerate.
(a) four (b) three (c) two (d) six

(2) The unperturbed wave functions of a particle in an infinite square well potential of bottom a are $\psi(x) = \left(\frac{2}{a}\right)^{\frac{1}{2}} sin(\frac{n\pi x}{a})$. If the bottom of the well is raised by an amount V_0 system is perturbed. What is the first order correction to the energy?
(a) 0 (b) $-V_0$ (c) V_0 (d) ∞

(3) Mathematical condition showing validity of WKB approximation is given by _____.
(a) $\frac{\hbar^2 S_2}{S_0} \ll 1$ (b) $\frac{\hbar^2 S_2}{S_0} \gg 1$ (c) $\frac{\hbar^2 A_2}{A_0} \ll 1$ (d) $\frac{\hbar^2 A_2}{A_0} \gg 1$

(4) The use of the Born approximation is best justified when _____. Here, K = kinetic energy, P = potential energy and E = total energy.
(a) K < P (b) K = E (c) K = P = 0 (d) K >> P

(5) The criterion to estimate the ground state energy by variation method is given by _____.
(a) $(W - E_0)^2 \leq [\langle H^2 \rangle_\psi - W^2]$ (b) $(W - E_0)^2 \geq [\langle H^2 \rangle_\psi - W^2]$
(c) $(W + E_0)^2 \leq [\langle H^2 \rangle_\psi - W^2]$ (d) $(W + E_0)^2 \geq [\langle H^2 \rangle_\psi - W^2]$

(6) Free particle Hamiltonian and its eigenfunction in one dimension are, respectively, given by $H = -\frac{\hbar^2}{2m}\frac{d^2}{dr^2}$ and $\varphi(r) = e^{-\alpha r}$ with α as a real constant. Expectation value of H is _____.
(a) α^2 (b) $-\frac{\hbar^2\alpha}{2m}$ (c) 0 (d) $-\frac{\hbar^2\alpha^2}{2m}$

(7) If $\hat{A}$ is any Hermitian operator and α is a real number, then _____ is *unitary*.
(a) $\hat{A}\alpha$ (b) $\hat{A}$ (c) exp $(i\alpha\hat{A})$ (d) $\hat{A}^\dagger$

(8) For linear operators $\hat{F}$ and $\hat{G}$, in A-representation, their product $[\hat{F}\hat{G}]_A$ is written as _____.
(a) $[\hat{G}\hat{F}]_A$ (b) $[\hat{F}]_A[\hat{G}]_A$ (c) $[\hat{G}]_A[\hat{F}]_A$ (d) $[\hat{F}\hat{G}]_A^{-1}$

Q.2 Answer any seven questions. **(14)**

(1) Show that the eigenvalues of a Hermitian operator are real.
(2) Define Green's function. $\nabla^2\left(\frac{1}{\vec{r}}\right) =$ _____.
(3) Give difference between the Born approximation and the partial wave analysis for scattering phenomena.
(4) What is Born series?
(5) Prove that the variation technique provides upper bound to the ground state energy.
(6) Why WKB approximation is also known as *semi-classical* approximation?

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(7) Define projection operator and unit operator.

(8) *Estimate* the ground state energy of one dimensional harmonic oscillator of mass m and angular frequency ω. Given: $\psi(x) = (\frac{2\alpha}{\pi})^{\frac{1}{4}} \cdot exp(-\alpha x^2)$ and $V(x) = \frac{1}{2}m\omega^2 x^2$.

(9) Write any two conditions to qualify any vector space as the *Hilbert* space.

Q.3 **(a)** What do we mean by Unitary transformation? Prove that the operator $(\frac{\hat{L}_z}{\hbar})$ plays the role of the generator of infinitesimal rotations. **(6)**

(b) Deduce the relation $(\chi)_A = [F]_A(\psi)_A$ for any linear operator $\hat{F}$. What is the matrix $[A]_A$ for operator $\hat{A}$ in its own representation? **(6)**

OR

(b) Prove that for a continuous basis, $\langle x|\hat{p}|x\rangle = -i\hbar\frac{\partial\psi(x)}{\partial x}$. **(6)**

Q.4 **(a)** Based on the method due to Dalgarno and Lewis, derive an expression for the second order correction to the ground state of H-atom and thus obtain an equation for polarizability. **(6)**

(b) Derive expressions for first excited energy levels of the H-atom using degenerate perturbation theory. **(6)**

OR

(b) What is meant by perturbation? Derive basic equations in various orders of perturbations. Derive expression for first order correction to energy eigenvalue. **(6)**

Q.5 **(a)** Within the WKB approximation, obtain the asymptotic (far from turning point) solution for one dimensional Schrödinger wave equation. **(6)**

(b) For the ground state of two-electron atom, assuming effective charge as a variational parameter, obtain $W_{min} = -\left(Z - \frac{5}{16}\right)\frac{e^2}{a_0}$. Here, a_0 is the Bohr radius. **(6)**

OR

(b) Discuss general technique to select trial wave function linear in variational parameters. Why variation technique yields better estimates of ground state energy then the perturbation technique? **(6)**

Q.6 **(a)** Using the method of Green's function technique, obtain the formal expressions for scattering amplitude, $f(\theta, \varphi)$. **(6)**

(b) (i) Derive mathematical condition for the validity of Born approximation for central potential. **(5)**

(ii) State optical theorem. **(1)**

OR

(b) What is Eikonal approximation? Derive an expression for scattering amplitude using the Eikonal approximation. **(6)**

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