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(16) **SARDAR PATEL UNIVERSITY**

M. Sc. Physics IInd Semester Examination

Saturday, Date: 18-04-2015 Time:10.30 A.M. to 01.30 P.M.

CBCS Course No.: PS02CPHY01

Subject: Quantum Mechanics–I

Note: Symbols have their usual meaning. **Total Marks: 70**

Q.1 Write answers of all eight questions in a table form by showing your choice against the question number. **(8)**

(1) The mathematical statement for the *Optical* theorem is _______.

(a) $\sigma > \frac{4\pi}{k} Im\, f(0)$ (b) $\sigma = \frac{4\pi}{k} Im\, f(0)$ (c) $\sigma < \frac{4\pi}{k} Im\, f(0)$

(d) $\sigma \leq \frac{4\pi}{k} Im\, f(0)$

(2) For any dynamical operator $\hat{A}$, the expectation value, $\langle \hat{A}^{\dagger}\hat{A}\rangle$ is ________.

(a) ≤ 0 (b) $= \infty$ (c) $= 0$ (d) ≥ 0

(3) The basic idea in Rayleigh–Schrödinger perturbation theory is to write the total Hamiltonian as a _______ of unperturbed and perturbed part of Hamiltonian.

(a) product (b) sum (c) ratio (d) difference.

(4) At the classical turning point; _________ .

(a) $E - V(x) = 0$ (b) $E - V(x) < 0$ (c) $E - V(x) > 0$ (d) $E - V(x) = \infty$

(5) If the eigen value E_m is non-degenerate, then $|v^{(0)}>$ _________ be defined uniquely.

(a) can (b) cannot (c) may (d) may not

(6) The first-order perturbation theory of a degenerate level is equivalent of finding _______ with respect to which the perturbation is diagonal.

(a) orthogonal ket vectors (b) normalized ket vectors (c) basis vectors

(d) null vectors

(7) The first order stark effect in the ground state of H-atom is __________.

(a) very large (b) zero (c) dependent on the magnitude of the electric field

(d) dependent on the square of magnitude of the electric field

(8) At large distance from the target, the scattered particles appear *radially outwards*, hence the scattered particles are represented as _______ waves.

(a) spherical (b) cylindrical (c) plane (d) spiral

Q.2 Answer any seven questions. **(14)**

(1) Show that the eigenvalues of a Hermitian operator are real.
(2) For hard-sphere scattering, write expression for phase shift δ_l, and interpret it.
(3) Give general technique to select *trial* wave function linear in variational parameters.
(4) Prove that the criterion to estimate the ground state energy using the variation technique is given by $[\langle\psi|H^2|\psi\rangle - W^2]^{\frac{1}{2}} \geq (W - E_0)$.
(5) Why WKB approximation is also known as *semi-classical* approximation?
(6) Prove that any observable is always diagonal in its own representation.

(7) Why variation technique yields better estimates of ground state energy then the perturbation technique?

(8) For anharmonic oscillator, taking $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2$ and $H' = bx^4$, obtain an expression for first-order correction to the energy $W^{(1)}$. Eigen ket corresponding to unperturbed part of the Hamiltonian (H_0) is given in terms of Hermite polynomial $H_n(\rho)$ as follows.

$|u_n\rangle = N_n \left[e^{-\frac{1}{2}\rho^2} H_n(\rho)\right]$. Here, $N_n = \left(\frac{\alpha}{2^n \sqrt{\pi}\, n!}\right)^{\frac{1}{2}}$, $\rho = \alpha x$, $\alpha = \left(\frac{m\omega}{\hbar}\right)^{\frac{1}{2}}$, and $k = m\omega^2$.

(9) Give difference between the Born approximation and the partial wave analysis for scattering phenomena.

Q.3 (a) Prove that for a continuous basis, $\langle x|\hat{p}|x\rangle = -i\hbar \frac{\partial\psi(x)}{\partial x}$. **(6)**

(b) What do we mean by Unitary transformation? Prove that the operator $(\frac{\hat{L}_z}{\hbar})$ plays the role of the generator of infinitesimal rotations. **(6)**

OR

(b) Deduce the relation $(\chi)_A = [F]_A(\psi)_A$ for any linear operator $\hat{F}$. How this representation differs from the Schrödinger representation? **(6)**

Q.4 (a) Using the technique due to Dalgarno and Lewis, prove that the *polarizability* (α) of the H-atom in its ground state is equal to $\frac{9}{2}a_0^3$, when it is kept in uniform external electric field. Here, a_0 represents Bohr radius. **(6)**

(b) Discuss the binding of two-electron atoms using the perturbation technique. **(6)**

OR

(b) Write basic assumptions involved in Raylaigh-Schrödinger perturbation theory. Write equations in various orders of perturbation. For non-degenerate case, obtain expressions for energy eigen value and eigen function corrected up to the first-order in perturbation. **(6)**

Q.5 (a) Describe the basic procedure involved in the variation technique. For the ground state of two-electron atom, assuming effective charge as a variational parameter, obtain $W_{min} = -\left(Z - \frac{5}{16}\right)\frac{e^2}{a_0}$. Here, a_0 is the Bohr radius. **(6)**

(b) Obtain the asymptotic solution for one dimensional Schrödinger equation using WKB approximation. **(6)**

OR

(b) Apply variation technique to obtain the ground state energy for H_2 molecule. Discuss importance of *overlap* and *exchange* interactions in binding of H_2 molecule. **(6)**

Q.6 (a) Define Green's function. Derive formal expression for scattering amplitude in terms of appropriate Green's function. **(6)**

(b) Discuss partial wave analysis, and obtain the expression for phase shift δ_l. **(6)**

OR

(b) What is the Born approximation? Obtain the expression for scattering amplitude within the Born approximation. Mention its validity. **(6)**

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