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## SARDAR PATEL UNIVERSITY

M. Sc. (Physics) (1<sup>st</sup> Semester) Examination

Day: Monday, Date: 24/10/2016, Time: 10:00 a.m. to 01:00 p.m.

Course No. PS01CPHY02 (Classical and Statistical Mechanics)

CBCS (choice based credit system)

**Important Note:** Signs have their usual meaning.

Q.1: Eight multiple choice questions (MCQ) carry one mark each.

Q.2: Short answer questions carrying two marks each (attempt any seven out of nine).

Q.3 to Q.6: Long answer questions carrying 12 marks each.

**Total Marks: 70**

- Q1 (i) In spherical coordinate system  $(r, \theta, \phi)$  if the Lagrangian  $L$  is given as  $L = \frac{1}{2}m[\dot{r}^2 + r\dot{\theta}^2 + r^2\dot{\phi}^2 \sin^2\theta]$ , then determine the cyclic coordinate; where mass  $m$  is a constant.  
 (a)  $r$  (b)  $m$  (c)  $\theta$  (d)  $\phi$
- (ii) For a transformation  $P_k = P_k(p, t)$ ,  $p$  and  $P_k$  are dependent variable than which generating function is excluded;  
 (a)  $F_1(q, Q, t)$  (b)  $F_2(q, P, t)$   
 (c)  $F_3(p, Q, t)$  (d)  $F_4(p, P, t)$
- (iii) The Poisson bracket  $[p_i, H]$  is equal to \_\_\_\_\_.  
 (a)  $-p_i$  (b)  $-\frac{\partial H}{\partial q_i}$  (c)  $p_i$  (d)  $q_i$
- (iv) The equilibrium is said to be stable if extremum value of potential energy  $V$  is \_\_\_\_\_.  
 (a) maximum (b) minimum  
 (c) zero (d) any value between minimum and maximum
- (v) The expectation value of any observable quantity  $\hat{F}$  in terms of density matrix is given as  
 (a)  $\text{Tr}(\hat{F}|\hat{\rho})$  (b)  $\hat{F}/\text{Tr}(\hat{\rho})$   
 (c)  $\hat{F}\text{Tr}(\hat{\rho})$  (d)  $\text{Tr}(\hat{\rho}\hat{F})$
- (vi) Ideal Fermi gas pressure at  $T = 0K$  is given by,  
 (a)  $3/2n\epsilon_F$  (b)  $1/2n\epsilon_F$  (c)  $n\epsilon_F$  (d)  $2/5n\epsilon_F$
- (vii) According to Clausius-Clapeyron equation, the change of vapour pressure with temperature is given as.  
 (a)  $\frac{1}{T(v_2 - v_1)}$  (b)  $\frac{L}{T(v_2 - v_1)}$  (c)  $\frac{T}{L(v_2 - v_1)}$  (d)  $\frac{L}{v_2 - v_1}$
- (viii) Order parameter is always \_\_\_\_\_ above critical point.  
 (a) 1 (b) 0 (c) 0.5 (d) 1.5
- Q2 (i) Discuss Poisson brackets of representing the equation of motion in a symmetric form. Prove Kronecker delta property?
- (ii) Explain with suitable examples stable and unstable equilibrium?
- (iii) What are the characteristics of a chaotic motion?
- (iv) Explain with suitable example the virtual displacement of a system.
- (v) Explain how the Hamilton-Jacobi equations are first-order partial differential equation in  $(n+1)$  variable.
- (vi) Write the density operators of various ensembles in the energy representation.
- (vii) Obtain the conduction electron density in terms of the Fermi energy  $\epsilon_F$  for ideal Fermi gas.

- (viii) Define phase transition and explain pressure versus temperature diagram of matter.
- (ix) Express the partition function for a real gas in terms of a two body interaction potential  $U(r_{ij})$ .

- Q3** (a) Solve the harmonic oscillator problem by canonical transformation. The given generating function for the harmonic oscillator is  $F_1 = \frac{1}{2}m\omega q^2 \cot Q$ . 6
- (b) What is gauge transformation? What arbitrariness does it introduces? Derive the new Lagrangian  $L'$  in terms of old Lagrangian  $L$  using generating function. 6

**OR**

- (b) Derive Hamilton-Jacobi equation for conserved Hamiltonian  $H$ . Discuss about Hamilton's principal function and show that it differs from the indefinite time integral of the Lagrangian by a constant term. 6
- Q4** (a) Explain small oscillations, stable and unstable equilibrium. Derive the equation of Lagrangian for the small oscillations near its equilibrium position. 6
- (b) Explain secular equation of oscillatory motion of a system. Solve the problem of two coupled simple pendulums and derive the equations of frequencies. 6

**OR**

- (b) Show that the eigenvectors corresponding to the two distinct eigenfrequencies are orthogonal. Explain the meaning of orthogonality. 6
- Q5** (a) Obtain the expression for the total number of particles in ideal Bose gas and discuss the Bose-Einstein condensation in detail. 6
- (b) Derive partition function and density matrix of free particle in the momentum representation. 6

**OR**

- (b) Write a note on white dwarf star and derive Chandrasekhar limit. 6
- Q6** (a) Express the following cluster integral terms in the perturbation expansion of an interacting system of eight particles diagrammatically and identify the number of  $l$ -particle clusters. 6
- (a)  $\int \dots \int f_{12}f_{35}f_{23}f_{78}d^3r_1 \dots d^3r_8$
- (b)  $\int \dots \int f_{12}f_{23}f_{13}f_{56}d^3r_1 \dots d^3r_8$
- (b) Discuss 1-D Ising model and its limitations. 6

**OR**

- (b) Derive Boltzmann transport equation and prove that  $\frac{dH}{dt} \leq 0$ . 6

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