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Sardar Patel University  
Vallabh Vidyanagar  
M Sc (Physics) - I Semester Examination  
PS01CPHY21 Mathematical Physics  
Day & Date: Tuesday, 19 March 2019

Time: 2:00 to 5:00pm

Max marks: 70

- I. Choose the best possible answer from the choices given below the questions (8x1=8)
- The norm of arrows lying in a plane is simply the  
(a) direction of the arrow (b) length of the arrow  
(c) plane in which the arrow lies (d) square of its length
  - If the operator X satisfying  $XA = E$ , where E is the identity operator then X is called  
(a) right inverse of A (b) left inverse of A  
(c) norm of A (d) dual operator of A
  - The only projection operator which has an inverse is the  
(a) null operator (b) vector operator  
(c) identity operator (d) adjoint operator
  - If there exist correspondence between the elements of the two representations of a group that satisfies the same group multiplication table, then the two representations are said to be  
(a) Cyclic (b) Homomorphic  
(c) Identical (d) Isomorphic
  - If  $f(z) = z^2$  and  $g(z) = z^*$  then it can be proved that  
(a) both  $f(z)$  and  $g(z)$  are analytic (b) both  $f(z)$  and  $g(z)$  are not analytic  
(c)  $f(z)$  is analytic  $g(z)$  is not (d)  $f(z)$  is not analytic  $g(z)$  is analytic
  - The Fourier transform of a Gaussian function is  
(a) another Gaussian with different width (b) same Gaussian with same width  
(c) a delta function (d) a polynomial
  - The Laplace transform of  $\cos(kx)$  is given by  
(a)  $s / (s^2 + k^2)$  (b)  $s^2 / (s^2 + k^2)$  (c)  $s / (s + k)$  (d)  $s / (s^2 - k^2)$
  - The Green's function corresponds to the Laplacian in three dimensional space is given by  
(a)  $4\pi(r_1 - r_2)$  (b)  $\frac{1}{(r_1 - r_2)^2}$  (c)  $(r_1 - r_2)^2$  (d)  $4\pi \frac{1}{(r_1 - r_2)}$

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(P.T.O)

II. Short answer questions ( Answer any seven questions given below. 7x2 = 14)

1. State the properties of a linear vector space
2. Define Hilbert space and discuss their properties
3. What are the defining properties of a group?
4. Show how the components of a contravariant and covariant vectors transform.
5. Write Laurent series. Define analytic function.
6. Obtain the Laplace transform of  $\cosh(kt)$
7. Write Cauchy- Riemann conditions. Show that for an analytic function  $f(z)$ , the derivative of  $f(z)$  with reference to  $z^*$  vanishes.
8. Write the properties of an analytic function.
9. Explain the defining properties of Green's function.

III A. Define an Eigen value equation. Show that the Eigen values of a Hermitian operator are all real and Eigen vectors corresponding to two different eigen values of a Hermitian operator are orthogonal. (6)

B. State and prove Cauchy- Schwarz inequality. (6)

OR

B. Define the following operators and state their properties i) adjoint operator, ii) Hermitian operator, iii) Projection operator. (6)

IV A. Find the Fourier transform of a finite wave train. Using the result derive the energy time uncertainty relationship. (6)

B. Using Laplace transform solve the LCR circuit. Obtain the resonance frequency of the circuit (6)

OR

B. Using the Fourier transform obtain the expression for the ground state of hydrogen atom given by

$$\varphi(\mathbf{r}) = \left(\frac{1}{\pi a_0}\right)^{1/2} \exp\left(-\frac{r}{a_0}\right) \text{ into momentum space. (6)}$$

V A. Derive the residue theorem and obtain an expression for the Cauchy Principal value. Evaluate the definite integral  $\int_0^{2\pi} \frac{d\theta}{1+\epsilon \cos\theta}$  (6)

B. Using Contour of integration, evaluate  $\int_0^{\infty} \frac{\sin x}{x} dx$  (6)

OR

B. Discuss various applications of Green's function. Derive a closed form of the Green's function corresponds to the Laplacian operator. (6)

VI A. Define the Christoffel symbol of first and second kind. Express the covariant derivatives of a second rank contravariant tensor. (6)

B. What are the reducible and irreducible representations of a group? Explain them with the help of the Group (i, -i, 1, -1). (6)

OR

B. Define character of a Group and explain its properties. (6)

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