(A-15) SEAT No.________

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SARDAR PATEL UNIVERSITY

M. Sc. (Semester III) Examination

Date: 3-11-2018, Saturday **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH25 – (Graph Theory – I)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) If $K_{1,n} = K_{n+1}$, then

(a) n = 1 (b) n = 2 (c) n > 2 (d) none of these

(2) A symmetric digraph is

(a) connected (b) balanced (c) regular (d) none of these

(3) Let T be a spanning in-tree with root R. Then

(a) $d^+(R) > 0, d^-(R) > 0$ (c) $d^+(R) = 0, d^-(R) > 0$

(b) $d^-(R) = 0, d^+(R) > 0$ (d) $d^+(R) = 0, d^-(R) = 0$

(4) If G is a simple digraph with vertices $\{v_1, v_2, \ldots, v_n\}$ & e edges, then $\sum_{i=1}^{n} d^+(v_i) =$

(a) ne (b) e^2 (c) 2e (d) e

(5) Which of the following graphs is Hamiltonian?

(a) $K_{3,5}$ (b) P_{49} (c) C_{50} (d) P_{50}

(6) The coefficient c_3 in chromatic polynomial of K_5 is

(a) 0 (b) 1 (c) 3 (d) 3!

(7) Let G be a simple graph without isolated vertex. Then a matching M in G is

(a) maximum $\Rightarrow$ perfect (c) maximal $\Rightarrow$ maximum

(b) maximal $\Rightarrow$ perfect (d) maximum $\Rightarrow$ maximal

(8) If $G = K_{2,n}$, then $\alpha(G) =$ ____.

(a) 2 (b) n (c) max{2, n} (d) min{2, n}

2. Attempt any SEVEN: [14]

(a) Find the diameter of $K_{2,3}$.

(b) Prove or disprove: A balanced digraph is regular.

(c) Define fundamental circuit matrix in a digraph.

(d) Prove: If G is a bipartite graph, then $\chi(G) = 2$.

(e) What is Four color problem?

(f) Define uniquely colourable graph.

(g) Define isomorphic graphs and give one example of it.

(h) Prove: If $S \subset V(G)$ is a vertex cover, then $V(G) - S$ is an independent set, in G.

(i) Define perfect matching in a graph and find it for K_8.

(P.T.O.)

3. (a) Define the following digraphs with examples: [6]
(i) Asymmetric (ii) complete asymmetric (iii) Symmetric (iv) complete symmetric

(b) Prove that if G is connected Euler digraph, then it is balanced. [6]

OR

(b) Obtain De Bruijn cycle for $r = 3$ with all detail. [6]

4. (a) Show that the determinant of every square sub matrix of incidence matrix A(G) of a digraph G is 1, –1 or 0. [6]

(b) Let A and B denote resp. the incidence matrix and circuit matrix of a digraph G without self-loop. Then prove that $AB^T = 0$. [6]

OR

(b) Define arborescence and show that an arborescence is a tree in which every vertex other than the root has an in – degree exactly one. [6]

5. (a) Prove: If G is Hamiltonian, then, for each $S \subset V(G)$, $c(G - S) \leq |S|$. [6]

(b) Prove: For a connected graph G, $\chi(G) = 2$ if and only if G has no odd cycle. [6]

OR

(b) Find the Chromatic polynomial of graph $K_{1,3}$. [6]

6. (a) State Hall's theorem and show that a k-regular bipartite graph has a perfect matching. [6]

(b) Let G be a graph (no isolated vertex) with n vertices. Prove that $\alpha'(G) + \beta'(G) = n$. [6]

OR

(b) Define $\alpha(G)$ and $\beta(G)$ and find it with corresponding sets, for $G = P_5$. [6]

x-x-x-x-x-x