(50) SEAT No.______

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester III) Examination

Date: 3 – 11 – 2018, Saturday **Time: 2.00 To 5.00** PM

Subject: MATHEMATICS **Paper No. PS03EMTH23 – (Graph Theory – II)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The number of spanning trees in $K_{1,n}$ is
(a) n (b) n! (c) n^n (d) 1

(2) The graph K_{11} can be decomposed into copies of
(a) $K_{1,6}$ (b) P_5 (c) P_6 (d) none of these

(3) A shortest path between two vertices in a graph can be obtained using
(a) Kruscal algorithm (c) BFS algorithm
(b) Dijkstra's algorithm (d) none of these

(4) If f is a flow on a network N = (V, A), then f(V,{t}) is
(a) f(V,{s}) (b) f({s},V) (c) f({t}, V) (d) none of these

(5) Let A be a matrix with spectrum {2, 1, –2, –3, –1}. Then det(A) =
(a) –3 (b) 3 (c) 12 (d) –12

(6) Let G = $K_{4,6}$. Then the non-zero eigen values for G is
(a) 2 (b) 4 (c) 6 (d) 12

(7) The Ramsey number R(3, 3) is
(a) 3 (b) 6 (c) 9 (d) none of these

(8) If E = {a, b, c} with M = {{a}, {b}, {a,b}} as hereditary system, then C_M =
(a) {c} (b) {{c}, {b,c}} (c) {{c}, {a,c}} (d) {{a,b,c}}

2. Attempt any SEVEN: [14]

(a) Find Pruffer code of all possible trees for which the degree sequence is (1,1,1,2,3).

(b) Give one graceful labelling of P_7 with detail.

(c) Define source and sink in a network.

(d) Define u-v edge separating set and give one example of it.

(e) Prove: If G is a simple graph with n vertices & e edges, then $2e(G) \leq n(G)\lambda_{max}(G)$.

(f) Prove: If G is k regular graph, then k is an eigen value of G.

(g) State Pigeonhole Principle.

(h) Prove: R(p, 2) = p, for every p ≥ 2.

(i) Prove: For $X \subset E$ and $e \in E$, $r(X + e) \leq r(X) + 1$.

(P.T.O)

3. (a) Prove that for a set $S \subseteq \mathbb{N}$ of size n, there are n^{n-2} trees with vertex set S. [6]

(b) How many trees are there with degree sequence (2,1,2,1,2)? Construct any one such tree. [6]

OR

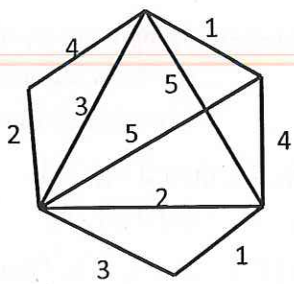
(b) Find $\tau(G)$ using Matrix-Tree theorem, for $G = C_5$.

4. (a) Let f be a flow in a network N = (V, A) with value d. Prove that, if $A(X, \overline{X})$ is a cut in N, then $d = f(X, \overline{X}) - f(\overline{X}, X)$. [6]

(b) Define flow and cut in a network and give one example of it. [6]

OR

(b) Using Kruskal's algorithm, find a shortest spanning tree for the graph below. [6]



5. (a) Prove: For any graph G, $\chi(G) \leq 1 + \lambda_{max}(G)$. [6]

(b) Prove: If H is an induced sub graph of G, then $\lambda_{max}(H) \leq \lambda_{max}(G)$. [6]

OR

(b) Prove: If G is bipartite graph, then n0n-zero eigen values of G occur in pair $(\lambda, -\lambda)$. [6]

6. (a) Prove: $R(p, q) \leq R(p-1, q) + R(p, q-1)$, $\forall$ p, q > 2. [6]

(b) Prove (ANY ONE): In a hereditary system, [6]

(i) Uniformity property (U) $\Rightarrow$ Sub modularity property (R)

(ii) Sub modularity property (R) $\Rightarrow$ Weak elimination property (C).

x-x-x-x-x-x