[233]

SEAT No.________

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SARDAR PATEL UNIVERSITY

M. Sc. (Semester III) Examination

Date: 26-10-2018, Friday

Time: 2.00 To 5.00 p.m.

Subject: MATHEMATICS

Paper No. PS03EMTH01 – (Functional Analysis II)

Total Marks: 70

Note: Throughout the paper, X and Y denote nlspaces.

1. Choose the correct option for each question: [8]

(1) If $x \in K^n$, then which of the following is true?
(a) $\|x\|_\infty \le \|x\|_1$ (b) $\|x\|_2 \le \|x\|_\infty$ (c) $\|x\|_1 \le \|x\|_\infty$ (d) $\|x\|_1 \le \|x\|_2$

(2) Every linear functional on X is continuous, if X =
(a) l^1 (b) $\mathbb{C}^n$ (c) C[0, 1] (d) none of these

(3) Let Y be a closed subspace of a nls X. Then X is a Banach space if and only if
(a) Y is a Banach space
(b) X/Y is a Banach space
(c) both Y and X/Y are Banach spaces
(d) none of these

(4) A map F: $(C^1[0,1], \|\cdot\|_\infty) \to (C[0,1], \|\cdot\|_\infty)$ defined by $F(x) = x'$ is
(a) not linear
(b) continuous but not linear
(c) linear but not continuous
(d) neither linear nor continuous

(5) Let A ∈ BL(X). Which of the following is true?
(a) $\sigma(A) \subset \sigma_e(A)$ (b) $\sigma(A) \subset \sigma_a(A)$ (c) $\sigma_a(A) \subset \sigma_e(A)$ (d) $\sigma_e(A) \subset \sigma(A)$

(6) For $x \in X$, let $j_x: X' \to K$ be defined by $j_x(f) = f(x)$. Then $\|j_x\|$ =
(a) 1 (b) $\|x\|$ (c) $\|f\|$ (d) none of these

(7) Let F ∈ BL(X,Y). Then
(a) $\|F\| < \|F'\|$ (b) $\|F\| > \|F'\|$ (c) $\|F\| = \|F'\|$ (d) none of these

(8) Let $\{x_n'\}$ be a sequence in X′ and $x' \in X'$. Which of the following is true?
(a) $x_n' \xrightarrow{w*} x' \Rightarrow x_n' \xrightarrow{\|\cdot\|} x'$
(b) $x_n' \xrightarrow{w} x' \Rightarrow x_n' \xrightarrow{\|\cdot\|} x'$
(c) $x_n' \xrightarrow{w*} x' \Rightarrow x_n' \xrightarrow{w} x'$
(d) $x_n' \xrightarrow{w} x' \Rightarrow x_n' \xrightarrow{w*} x'$

2. Attempt any SEVEN: [14]

(a) Prove: If $E \subset X$ is convex, then E^0 is convex.
(b) If X is an infinite dimensional nls, then show that there is linear map from X to K which is not continuous.
(c) State Uniform Boundedness Principle.
(d) Show that $\mathbb{R}^2$ with $\|(x_1, x_2)\|_1 = |x_1| + |x_2|$, is not strictly convex.
(e) Prove: If F: X→Y is continuous, then F is a closed map.
(f) Let X be a Banach space. If a series $\sum_n x_n$ of elements of X is absolutely summable, then show that it is summable in X.
(g) Define $\sigma_e(A)$, $\sigma_a(A)$ and $\sigma(A)$.
(h) Prove: If $F, G \in BL(X,Y)$, then $(F+G)' = F' + G'$.
(i) Define weak and weak* convergence in X′.

(PTO)

3. (a) State and prove Holder's inequality. [6]

(b) Prove: For $f \in C[a,b]$, $\|f\|_\infty = \{|f(t)| : t \in [a,b]\}$ defines a norm on $C[a,b]$. [6]

OR

(b) Let $F \in BL(X,Y)$. Define a map $\widetilde{F} : X/Z(F) \rightarrow Y$ by $\widetilde{F}(x + Z(F)) = F(x)$. Show that $\widetilde{F}$ is linear and continuous. [6]

4. (a) Prove: If Y is a Banach space, then $BL(X,Y)$ is complete. [6]

(b) Let X & Y be nlspaces and $F: X \rightarrow Y$ be linear. Prove that F is continuous if and only if goF is continuous, for every $g \in Y'$. [6]

OR

(b) State and prove Hahn-Banach separation theorem. [6]

5. (a) Prove: If X and Y are Banach spaces and $F: X \rightarrow Y$ is a closed linear map, then F is continuous. [6]

(b) Let $F: X \rightarrow Y$ be linear map. Prove that if there exists $\gamma > 0$ such that for every $y \in Y$, there is $x \in X$ with $F(x) = y$ and $\|x\| \leq \gamma \|y\|$, then F is an open map. [6]

OR

(b) Let $A \in BL(X)$ and $\dim X = \infty$. Show that $\sigma_e(A) = \sigma(A)$. [6]

6. (a) Prove: If X' is separable, then X is separable. [6]

(b) Let X be a finite dimensional space with $\dim X = m$ and let $\{a_1, a_2, \ldots, a_m\}$ be a basis for X. Show that $\dim X' = m$. [6]

OR

(b) Let X be a separable nls. Prove that every bounded sequence in X' has a weak* convergent subsequence. [6]

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