[211] SEAT No.______ **No. of Printed Pages : 3**

Sardar Patel University

M.Sc. (Mathematics) (Semester-III); Examination 2018

PS03CMTH22: Mathematical Methods-I

Date: 24th October, 2018, Wednesday Full Marks: 70

Time: 2:00 pm to 5:00 pm

Instructions:

1. **Attempt all questions.**
2. **Assume usual/standard notations wherever applicable.**
3. **Figures to the right indicate full marks.**

Q- 1 Choose the most appropriate option for each of following question: **[8]**

(1) The inverse Laplace transform of $\dfrac{Ke^{-as}}{s^2+k^2}$

(a) $\sin kt$ (b) $\cos kt$ (c) $K\cos kt$ (d) none of these

(2) The inverse Laplace transform of $\dfrac{1}{s(s^2+1)}$

(a) $1-\sin t$ (b) $1+\cos t$ (c) $1-\cos t$ (d) $1+\sin t$

(3) Find $L\left\{t^{-\frac{1}{2}}\right\}=$ ______

(a) $\dfrac{2}{\pi}s$ (b) $\sqrt{\dfrac{\pi}{s}}$ (c) πs (d) $\dfrac{\pi}{2}s$

(4) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}=$ ______

(a) $\dfrac{\pi^2}{6}$ (b) $\dfrac{2}{\pi}$ (c) π (d) none of these

(5) $Z\left\{\dfrac{x^n}{n!}\right\}=$ ______

(a) $\exp\left(\dfrac{x}{z}\right)$ (b) exp (x) (c) exp(z) (d) $\exp\left(\dfrac{z}{x}\right)$

(6) Fourier coefficient a_0 of the Fourier series of 2π periodic function $f(x)=1,\ -\pi<x\le\pi$ is ______

(a) 0 (b) 2 (c) 1 (d) 0.5

(7) $Z\{(-1)^n\}=$ ______

(a) $\dfrac{1}{z+1}$ (b) $\dfrac{z}{z-1}$ (c) $\dfrac{z}{z+1}$ (d) $\dfrac{1}{z-1}$

(8) For $a \neq 0$ then Fourier transform of $f(ax)$ is

(a) $F(s-a)$ (b) $\frac{1}{a}F\left(\frac{a}{s}\right)$ (c) $e^{-isa}F(s)$ (d) $\frac{1}{a}F\left(\frac{s}{a}\right)$

Q- 2 Attempt **any Seven** **[14]**

(a) Find $L^{-1}\{\cot^{-1}(1+s)\}$

(b) State Dirichlet theorem for the convergence of Fourier series.

(c) State Initial value theorem for Z-transform and verify it for the function

$$\frac{z}{(z-a)(z-b)}$$

(d) Find $L\{1 * e^{t}\}$

(e) State and prove Parseval's Identity for Fourier transform

(f) Prove that $L\{t^{n} f(t)\} = (-1)^{n} \frac{d^{n}}{ds^{n}}[L\{f(t)\}]$

(g) Let $a \in R$ and $f(x) \in L^{1}(R)$. Define $T_{a}f(x) = f(x-a)$ then prove that $F[T_{a}f] = e^{-ias}F[f]$

(h) Find the inverse Z transform of $\frac{z}{z^{2}-6z+8}$

(i) Solve the Initial value problem for the difference equation $f(n+1) - f(n) = 1,\ f(0) = 0$

(j) Let $f(x) = 1 + \cos 2x - \sin^{2} x$ be a 2π periodic function. Then find Fourier coefficient a_0 of $f(x)$.

Q-3 (a) Find the Fourier series for 2π periodic function $f(x) = \begin{cases} 0; & -\pi < x < 0 \\ \pi; & 0 < x < \pi \end{cases}$ **[6]**

(b) Find the Fourier series corresponding to the function $f(x)$ defined in $(-2, 2)$

$$f(x) = \begin{cases} 2; & -2 \le x \le 0 \\ x; & 0 < x < 2 \end{cases}$$ **[6]**

OR

(b) Compute the Fourier series of a 2π periodic function $f(x) = x^{2}$ hence find $\sum_{n=1}^{\infty} \frac{1}{n^{4}}$

Q-4 (a) If $L\{f(t)\} = F(s)$ then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_{s}^{\infty} F(s)\,ds$ and hence evaluate **[6]**

$$L\left[e^{-4t}\frac{\sin 3t}{t}\right]$$

(b) Solve $u_{xx} = \frac{1}{c^2} u_{tt} + k, \quad 0 < x < l, \; t > 0$ subject to $u(0,t) = u_x(0,t) = 0$ for all $t > 0$ and $u(x,0) = u_t(x,0) = 0$ for all x. **[6]**

OR

(b) Using Error function, evaluate $L^{-1}\left\{\frac{1}{\sqrt{s}(s-a)}\right\}$ and hence find $L^{-1}\left\{\frac{1}{\sqrt{s}-\sqrt{a}}\right\}$

Q-5 (a) If $x^n f(x) \in L^1(R)$ then obtained its Fourier transform and hence compute $F\left\{x^2 \exp\left(-ax^2\right)\right\}$ **[6]**

(b) Find ϕ if $\int_{-\infty}^{\infty} \phi(t) \exp\{-|x-t|\} dt = x^3$ **[6]**

OR

(b) Solve $u_t = k u_{xx} \quad (x, t > 0)$ subject to $u(x,0) = 0$ and $u_x(0,t) = -a$ also both $u, u_x \to 0$ as $x \to \infty$

Q-6 (a) Define Z-transform and its Convolution. State and prove Convolution theorem for Z transform and hence **[6]**

Compute: (i) $Z^{-1}\left[\frac{z^2}{(z-2)(z-5)}\right]$ (ii) $Z^{-1}\left[\frac{z^2}{(z-3)^2}\right]$

(b) (i) State and prove Final value theorem for Z-transform. **[3]**

(ii) If $Z\{f(n)\} = F(z)$ then show that $Z\{n a^n f(n)\} = -z \frac{d}{dz}\left\{F\left(\frac{z}{a}\right)\right\}$ **[3]**

OR

(b) If $Z\{f(n)\} = F(z)$ then show that

(i) $\sum_{k=0}^{n} f(k) = Z^{-1}\left\{\frac{z}{z-1} F(z)\right\}$ **[3]**

(ii) $Z\left\{\frac{f(n)}{n+m}\right\} = z^m \int_z^{\infty} \frac{F(\xi) d\xi}{\xi^{m+1}}$ and hence find $Z\left\{\frac{1}{n+1}\right\}$ **[3]**

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