M.Sc. (Mathematics) (Semester-III); Examination 2018

PS03CMTH02: Mathematical Methods-I

Date: 24th October, 2018, Wednesday

Full Marks: 70

Time: 2:00 pm to 5:00 pm

Instructions:

- 1. Attempt all questions.
- 2. Assume usual/standard notations wherever applicable.
- 3. Figures to the right indicate full marks.
- Choose the most appropriate option for each of following question:

[8]

- The inverse Laplace transform of $\frac{Ke^{-as}}{s^2 + k^2}$ **(1)**
 - (a) $\sin kt$
- (b) cos *kt*
- (c) $K \cos kt$ (d) none of these
- (2) The inverse Laplace transform of $\frac{1}{s(s^2+1)}$

- (a) $1-\sin t$ (b) $1+\cos t$ (c) $1-\cos t$ (d) $1+\sin t$
- (3) Find $L\left\{t^{-\frac{1}{2}}\right\} = \underline{\hspace{1cm}}$
 - (a) $\frac{2}{\pi}s$ (b) $\sqrt{\frac{\pi}{s}}$ (c) πs (d) $\frac{\pi}{2}s$

- (4) $\sum_{1}^{\infty} \frac{1}{n^2} = \underline{\hspace{1cm}}$
 - (a) $\frac{\pi^2}{6}$ (b) $\frac{2}{\pi}$ (c) π
- (d) none of these

- $(5) Z\left\{\frac{x^n}{n!}\right\} = \underline{\hspace{1cm}}$
 - (a) $\exp\left(\frac{x}{z}\right)$ (b) $\exp(x)$ (c) $\exp(z)$ (d) $\exp\left(\frac{z}{x}\right)$

- (6) Fourier coefficient a_0 of the Fourier series of 2π periodic function
- $f(x)=1, -\pi < x \le \pi \text{ is}$ _____ (a) 0 (b) 2 (c) 1 (7) $Z\{(-1)^n\}=$ _____
- (d) 0.5

- - (a) $\frac{1}{z+1}$ (b) $\frac{z}{z-1}$ (c) $\frac{z}{z+1}$ (d) $\frac{1}{z-1}$

(8) For $a \neq 0$ then Fourier transform of f(ax) is

(a)
$$F(s-a)$$
 (b) $\frac{1}{a}F(\frac{a}{s})$ (c) $e^{-isa}F(s)$ (d) $\frac{1}{a}F(\frac{s}{a})$

Q-2 Attempt any Seven

[14]

- (a) Evaluate $L^{-1}\left\{\cot^{-1}(1+s)\right\}$
- (b) State Dirichlet theorem for the convergence of Fourier series.
- (c) Find $H_2(x)$
- (d) Obtain $L\{1*e^t\}$
- (e) State and prove Parseval's Identity for Fourier transform
- (f) Prove that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} [L\{f(t)\}]$
- (g) Let $a \in R$ and $f(x) \in L^1(R)$. Define $T_a f(x) = f(x-a)$ then prove that $F[T_a f] = e^{-ias} F[f]$
- (h) Find the inverse Z transform of $\frac{z}{z^2 6z + 8}$
- (i) Solve the Initial value problem for the difference equation f(n+1)-f(n)=1, f(0)=0
- (j) Let $f(x)=1+\cos 2x-\sin^2 x$ be a 2π periodic function. Then find Fourier coefficient a_0 of f(x).

Q-3 (a) Find the Fourier series for
$$2\pi$$
 periodic function $f(x) = \begin{cases} 0; & -\pi < x < 0 \\ \pi; & 0 < x < \pi \end{cases}$ [6]

(b) Find the Fourier series corresponding to the function f(x) defined in (-2, 2)

$$f(x) = \begin{cases} 2; & -2 \le x \le 0 \\ x; & 0 < x < 2 \end{cases}$$
 [6]

OR

(b) Compute the Fourier series of a 2π periodic function $f(x) = x^2$ hence find $\sum_{n=1}^{\infty} \frac{1}{n^4}$

Q-4 (a) If
$$L\{f(t)\}=F(s)$$
 then prove that $L\left\{\frac{f(t)}{t}\right\}=\int_{s}^{\infty}F(s)ds$ and hence evaluate
$$L\left[e^{-4t}\frac{\sin 3t}{t}\right]$$
[6]

$$y'' + 9y = \cos 2t$$
, $y(0) = 1$ and $y(\frac{\pi}{2}) = 1$

OR

[6]

(b) Show that:
$$L^{-1}\left\{\frac{s}{s^4+s^2+1}\right\} = \frac{2}{\sqrt{3}} \sin\left(\frac{\sqrt{3}}{2}t\right) \sinh\left(\frac{t}{2}\right)$$

Q-5 (a) If
$$x^n f(x) \in L^1(R)$$
 then obtained its Fourier transform and hence compute
$$F\left\{x^2 \exp\left(-ax^2\right)\right\}$$

(b) Find
$$\phi$$
 if $\int_{-\infty}^{\infty} \phi(t) \exp\{-|x-t|\}dt = x^3$ [6]

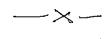
(b) Solve
$$u_t = ku_{xx}$$
 $(x,t>0)$ subject to $u(x,0)=0$ and $u_x(0,t)=-a$ also both $u,u_x\to 0$ as $x\to \infty$

Q-6 (a) Find Green's function for
$$y'' = f(x)$$
, $y(0) = y(1) = 0$ hence find its particular solution for $f(x) = x^2$

(b) Define Inner product space. Orthonormalize the set
$$\{1, x, x^2\}$$
 over $[-1, 1]$ [6]

OR

(b) Define Z transform and prove its any one property. Using Z transform, find 50th term of Fibonacci sequence.



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