

(85 & A-25) Seat NO: \_\_\_\_\_

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Sardar Patel University  
Mathematics

M.Sc. Semester III

Wednesday, 19 October 2016

2.00 p.m. to 5.00 p.m.

PS03CMTH02 - Mathematical Methods I

Maximum Marks: 70

Q.1 Choose the correct option for each of the following.

[8]

(1) Let  $f(x) = \cos x + i \sin x$ . Then the complex Fourier coefficient  $c_{-1}$  of  $f$  is

- (a) 1                      (b)  $i$                       (c)  $-i$                       (d) 0

(2) Let  $f$  be a  $2\pi$ - periodic function given by  $f(x) = x - x^2$ ,  $-\pi < x \leq \pi$ . The Fourier series of  $f$  at  $\pi$  converges to

- (a)  $-\pi^2$                       (b)  $\pi^2$                       (c)  $-\pi$                       (d)  $\pi$

(3) If  $f \in L^1(\mathbb{R})$  and  $F[f](s) = \frac{\sin 4s}{3s}$ ,  $s \neq 0$ , then  $F[f](0)$  is equal to

- (a)  $\frac{4}{3}$                       (b)  $\frac{3}{4}$                       (c) 1                      (d) 0

(4) Let  $f$  be an even integrable function. Which of the following is not true?

- (a)  $F[f] = F^{-1}[f]$       (b)  $F_c[f] = F[f]$       (c)  $F[f] = -iF_s[f]$       (d)  $F[F[f]] = F[f]$

(5)  $L[e^{at} \sinh bt](t) =$

- (a)  $\frac{a}{(s-a)^2 + b^2}$       (b)  $\frac{b}{(s-a)^2 + b^2}$       (c)  $\frac{a}{(s-a)^2 - b^2}$       (d)  $\frac{b}{(s-a)^2 - b^2}$

(6)  $L^{-1}\left[\frac{L[f](s)}{s}\right](t) =$

- (a)  $\int_t^\infty L[f](u) du$       (b)  $\int_0^t L[f](u) du$       (c)  $\int_t^\infty f(u) du$       (d)  $\int_0^t f(u) du$

(7) The Z- transform of  $(\frac{1}{n!})_{n \geq 0}$  is

- (a)  $e^z$                       (b)  $e^{-z}$                       (c)  $e^{\frac{1}{z}}$                       (d)  $e^{-\frac{1}{z}}$

(8) The value of  $\frac{d^{11}}{dx^{11}}\{e^{x^2} \frac{d^7}{dx^7}(e^{-x^2})\}$  at  $x = 0$  is

- (a) 7                      (b) 11                      (c)  $\frac{7}{11}$                       (d) none of these

Q.2 Attempt any Seven.

[14]

- (a) Compute the half range Fourier cosine series of  $f(x) = 1$ ,  $0 < x < \pi$ .  
(b) Express the complex Fourier coefficient  $c_n$  of a  $2\pi$ - periodic function  $f$  in terms of its Fourier coefficients  $a_n$  and  $b_n$ .  
(c) Evaluate  $\int_0^\infty \frac{\sin^2 x}{x^2} dx$  using Fourier transform methods.  
(d) Let  $u(x, t)$  be a function of two variables such that both  $u(x, t)$  and  $u_x(x, t)$  tend to 0 as  $x \rightarrow \infty$ . In usual notations show that  $F_s[u_{xx}](s) = -s^2 F_s[u](s) + \sqrt{\frac{2}{\pi}} s u(0, t)$ .

1

(1)

(P.T.O.)

- (e) Evaluate the Laplace transform of  $t^2 e^{-4t}$ .  
 (f) Find the inverse Laplace transform of  $\frac{e^{-\pi s}}{s^2+1}$ .  
 (g) Compute  $H_3(1)$ .  
 (h) Compute the inverse Z- transform of  $\frac{z}{(z-2)(z-3)}$ .  
 (i) State Gram-Schmidt Orthonormalization Theorem.

Q.3

- (a) Compute the Fourier series of 2- periodic function  $f(x) = x - x^2$ ,  $-1 < x \leq 1$ . Hence [6]  
 evaluate the sums of  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  and  $\sum_{n=1}^{\infty} \frac{1}{n^4}$   
 (b) Compute the half range Fourier cosine series of the function  $f(x) = \frac{2x}{\pi}$  if  $0 < x < \frac{\pi}{2}$  and [6]  
 $f(x) = \frac{2}{\pi}(\pi - x)$  if  $\frac{\pi}{2} \leq x < \pi$ . Hence evaluate  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ .

OR

- (b) Solve  $u_t = c^2 u_{xx}$ ,  $0 < x < 100$ ,  $t > 0$  subject to  $u(0, t) = u(100, t) = 0$ ,  $t \geq 0$ ,  $u(x, 0) = x$  [6]  
 for  $0 \leq x \leq 50$  and  $u(x, 0) = 100 - x$  for  $50 \leq x \leq 100$ . (you may take both functions equal  
 to  $-k^2$  while applying separation of variables)

Q.4

- (c) Solve  $u_{tt} = c^2 u_{xx}$ ,  $x, t > 0$  subject to  $u(x, 0) = f(x)$ ,  $u_t(x, 0) = g(x)$  for all  $x > 0$  and [6]  
 $u(0, t) = 0$  for all  $t > 0$  using Fourier transform methods.  
 (d) Compute the Fourier transform of  $\chi_{[-1,1]}$ . Hence evaluate the integrals  $\int_0^{\infty} \frac{\sin x \cos 2x}{x} dx$  and [6]  
 $\int_0^{\infty} \frac{\sin x}{x} dx$ .

OR

- (d) (a) If  $f, g \in L^1(\mathbb{R})$ , then show that  $f * g \in L^1(\mathbb{R})$ . [3]  
 (b) Let  $\alpha > 0$  and  $\alpha \neq 1$ . Solve  $y'' - y = e^{-\alpha|x|}$  subject to  $y(x) \rightarrow 0$ ,  $y'(x) \rightarrow 0$  as  $|x| \rightarrow \infty$ . [3]

Q.5

- (e) Use Laplace transform methods to solve  $u_{tt} = c^2 u_{xx}$ ,  $x > 0, t > 0$  subject to  $u(0, t) = [6]  
 A \sin wt$ ,  $u_x(0, t) = 0$  for all  $t$  and  $u(x, 0) = u_t(x, 0) = 0$  for all  $x$ .  
 (f) Compute the inverse Laplace transforms of the function  $\frac{s}{s^4+s^2+1}$  and  $\frac{e^{-3s}}{(s-1)^2+2}$ . [6]

OR

- (f) Solve  $y'' + 2y' + 5y = e^t \sin t$  subject to  $y(0) = 3$  and  $y'(0) = 1$ .

Q.6

- (g) Orthonormalize the set  $\{1, x, x^2\}$  over  $[-1, 1]$  and show that [6]

$$\text{sp}\{1, x, x^2, \dots\} = \text{sp}\left\{\sqrt{\frac{2n+1}{2}} P_n(x) : n \in \mathbb{N} \cup \{0\}\right\}.$$

- (h) (a) Solve  $y_{n+2} - 6y_{n+1} + 5y_n = 0$ ,  $n \geq 0$ , subject to  $y_0 = 1$  and  $y_1 = -2$ . [3]  
 (b) Find the Green's function for  $y''(x) + 9y(x) = f(x)$  subject to  $y(0) = 0$ ,  $y'(\pi) = 0$  and [3]  
 hence find the solution of the above equation where  $f(x) = x \cos x$ .

OR

- (h) (a) Find a polynomial of degree 2 so that  $\int_{-1}^1 |\sin x - p(x)|^2 dx$  is minimum. [3]  
 (b) Show that  $H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0$  for all  $n \geq 1$  and hence show that [3]  
 $H_n(x)$  satisfies  $y'' - 2xy' + 2ny = 0$ .

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