[177] SEAT No.________ No of printed pages: 2

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M.Sc. (Sem-III), PS03EMTH37, Mathematical Probability Theory;
Friday, 29^{th} March, 2019; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. Let $(\Omega, \mathcal{A}, P)$ be probability space. Which one from following is true ?
   (A) $P(A \cap B) = P(A) + P(B) + P(A \cup B)$
   (B) $P(A \cap B) - P(A) = P(A \cup B) - P(B)$
   (C) $P(A) - P(A \cap B) = P(A \cup B) - P(B)$
   (D) None of these
2. Let $P(A) = 0.30, P(B) = 0.78, P(A \cap B) = 0.16$. Then
   (A) $P(A^c \cap B^c) = 0.24$ (B) $P(A^c \cup B^c) = 0.76$
   (C) $P(A \cap B^c) = 0.14$ (D) None of these are true
3. Let X be a random variable on $(\Omega, \mathcal{A}, P)$. Then $P(X \leq a)$=
   (A) $\int_{-a}^{\infty} f(x)dx$ (B) $\int_{0}^{\infty} f(x)dx$ (C) $\int_{-\infty}^{a} f(x)dx$ (D) None of these
4. $\text{Var}(5 - X) =$
   (A) $\text{Var}(-X)$ (B) $5 - \text{Var}(X)$ (C) $\text{Var}(X)$ (D) $5 + \text{Var}(X)$
5. Let X_n be a sequence of rvs having $E(X_n) = 0$ and $Var(X_n) = \frac{2}{n}$. Then
   (A) $X_n \xrightarrow{P} 2$ (B) $X_n \xrightarrow{P} 1$ (C) $X_n \xrightarrow{P} 0$ (D) none of these
6. $X_n \xrightarrow{P} 0 \Leftrightarrow$
   (A) $\lim_{n\to\infty} E(X_n) = 0$ (B) $\lim_{n\to\infty} Var(X_n) = 0$
   (C) $\lim_{n\to\infty} Var(X_n^2) = 0$ (D) none of these
7. If $\phi(u)$ is characteristic function of random variable X, then
   (A) $\phi(-u) > \phi(0)$ (B) $\phi(-u) > \overline{\phi}(u)$
   (C) $\phi(-u) = \overline{\phi}(u)$ (D) none of (A),(B),(C) is true
8. Let F be a distribution function and h be corresponding characteristic function. For any $u > 0$, $\exists K > 0 \ni \frac{u}{K} \int_{|x| \geq \frac{1}{u}} dF(x)$
   (A) $\leq \int_0^u [Re(h(v)) - h(0)]dv$ (B) $\geq \int_0^u [h(0) - Re(h(v))]dv$
   (C) $\leq \int_0^u [h(0) - Re(h(v))]dv$ (D) none of these

Q.2 Attempt any ***seven***: [14]

(a) Define probability measure.
(b) Let $\Omega = \{a, b, c, d\}, \mathcal{A} = \{\phi, \Omega, \{a, b\}, \{c, d\}\}$ and $X : \Omega \to \mathbb{R}$ defined by $X(a) = -1 = X(b), X(c) = 1, X(d) = -2$. Is X a random variable?
(c) Show that sum of two rvs is a rv.
(d) State and prove Chebyshev's inequality.
(e) Show that $E(aX) = aE(X)$ where $a \in \mathbb{R}$ and X is non negative random variable.
(f) Define convergence almost surely.
(g) State monotone convergence theorem.
(h) State Helly-Bray theorem.
(i) Define characteristic function of a random variable.

(P.T.O.)

Q.3 [6]

(a) State and prove Jordan decomposition theorem.

(b) Let $X = (I_A, I_B)$ where $A, B \in \mathcal{A}$. Then Show that X is a vector random variable. [6]

OR

(b) Find mean and variance of standard normal random variable.

Q.4 [6]

(a) State and prove C_r inequality. [6]

(b) State and prove Jensen's inequality.

OR

(b) Let X & Y be simple rvs. Then show that $E(aX + bY) = aE(X) + bE(Y), a, b \in \mathbb{R}$.

Q.5 [6]

(a) Prove: $X_n \xrightarrow{P} 0 \Leftrightarrow E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$ as $n \to \infty$. [6]

(b) Prove: $X_n Y_n \xrightarrow{a.s.} XY$, if $X_n \xrightarrow{a.s.} X$ and $Y_n \xrightarrow{a.s.} Y$

OR

(b) State and prove dominated convergence theorem.

Q.6 [6]

(a) State and prove Levy's theorem. [6]

(b) State and prove Kolmogorav's inequality.

OR

(b) Show that characteristic function of random variable is continuous. State results which you use.

X

(2)