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SARDAR PATEL UNIVERSITY

M. Sc. (Semester III) Examination

Date: 1-4-2019, Monday **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH23 – (Graph Theory – II)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The number of spanning trees in $K_{1,4}$ is
(a) 4^3 (b) 4^2 (c) 4 (d) 1

(2) If all the digits in the Pruffer code are same, then the graph is
(a) Star graph (b) Path graph (c) Cycle graph (d) $K_{n,n}$ $(n > 1)$

(3) A shortest path between two vertices in a graph can be obtained using
(a) Kruscal algorithm (b) Dijkstra's algorithm
(c) BFS algorithm (d) none of these

(4) In a network, if s is source and t is sink, then
(a) $d^+(s) = 0 = d^-(t)$ (b) $d^+(s) > 0, d^-(t) > 0$
(c) $d^+(s) = 0, d^-(t) > 0$ (d) $d^+(s) > 0, d^-(t) = 0$

(5) Let A be a matrix with spectrum $\{-2, -1, 2, -3, 1\}$. Then det(A) =
(a) –12 (b) 12 (c) –2 (d) –3

(6) Let G be a graph with $\lambda_{max}(G) = 4$. Then $\chi(G)$
(a) ≥ 5 (b) ≤ 5 (c) ≤ 3 (d) ≥ 3

(7) The Ramsey number R(3, 3) is
(a) 3 (b) < 6 (c) > 6 (d) 6

(8) If E = {a, b, c} with $M = \{\{a\}, \{b\}, \{a,b\}\}$ as hereditary system, then C_M =
(a) {{c}, {a,c}} (b) {{c}, {b,c}} (c) {c} (d) {{a,b,c}}

2. Attempt any SEVEN: [14]

(a) Find a tree with Pruffer code (242).

(b) State Matrix tree theorem.

(c) If f is a flow on a network N = (V, A), then find f({s}, V) and f({t}, V).

(d) Define u-v vertex separating set and give one example of it.

(e) Prove: For any graph G, $\lambda_{max}(G) \leq \Delta(G)$.

(f) Prove: If G is k regular graph, then k is an eigen value of G.

(g) Prove: $R(p, 2) = p$, for every $p \geq 2$.

(h) State Pigeonhole Principle.

(i) Prove: For $X \subset E$ and $e \in E$, $r(X + e) \leq r(X) + 1$.

(P.T.O.)

3. (a) Prove: If a tree T with m edges has a graceful labelling, then K_{2m+1} can be decomposed into 2m+1 copies of T. [6]

(b) Find $\tau(C_4)$ by contraction of edge method. [6]

OR

(b) Prove: If $e \in E(G)$ is not a loop, then $\tau(G) = \tau(G - e) + \tau(G \bullet e)$. [6]

4. (a) Let f be a flow in a network N = (V, A) with value d. Prove that, if $A(X, \overline{X})$ is a cut in N, then $d = f(X, \overline{X}) - f(\overline{X}, X)$. [6]

(b) Define source, sink and flow in a network N and illustrate these concepts by giving one example of a network N with at least five vertices. [6]

OR

(b) Write Dijkstras's algorithm. [6]

5. (a) Prove: If G is bipartite graph, then non-zero eigen values of G occur in pair $(\lambda, -\lambda)$. [6]

(b) Find $sp(K_{1,3})$. [6]

OR

(b) Prove: The diameter of G is less than the number of distinct eigen values of G. [6]

6. (a) Prove: $R(p, q) \leq R(p-1, q) + R(p, q-1), \forall\ p, q > 2$. [6]

(b) Prove (ANY ONE): In a hereditary system, [6]
(i) Sub modularity property (R) $\Rightarrow$ Weak elimination property (C).
(ii) Augmentation property (I) $\Rightarrow$ Uniformity property (U).

X-X-X-X-X-X