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SEAT No.________

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SARDAR PATEL UNIVERSITY

M. Sc. (Semester III) Examination

Date: 25-3-2019, Monday **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH01 – (Functional Analysis II)**

Total Marks: 70

Note: Throughout the paper, X and Y denote nlspaces.

1. Choose the correct option for each question: [8]

(1) If $x \in K^n$, then which of the following is true?
(a) $\|x\|_2 \leq \|x\|_1$ (b) $\|x\|_1 \leq \|x\|_2$ (c) $\|x\|_1 \leq \|x\|_\infty$ (d) $\|x\|_\infty = \|x\|_1$

(2) Let Y be a subspace of X. Which of the following is true?
(a) If $Y \neq X$, then $Y^0 \neq \emptyset$ (b) If $Y \neq X$, then $Y^0 = \emptyset$
(c) If $Y^0 \neq \emptyset$, then $Y = X$ (d) If $Y = X$, then $Y^0 = \emptyset$

(3) Let Y be a closed subspace of a nls X. Then X is a Banach space if and only if
(a) Y is a Banach space (b) both Y & X/Y are Banach spaces
(c) X/Y is a Banach space (d) none of these

(4) A map F: $(C^1[0,1], \|.\|_\infty) \rightarrow (C[0,1], \|.\|_\infty)$ defined by $F(x) = x'$ is
(a) closed and continuous (b) continuous but not closed
(c) closed but not continuous (d) none of these

(5) Let $A \in BL(X)$. Which of the following is true?
(a) $\sigma(A) \subset \sigma_e(A)$ (b) $\sigma_e(A) \subset \sigma_a(A)$ (c) $\sigma_a(A) \subset \sigma_e(A)$ (d) none of these

(6) If $dim\, X = n$, then $dim\, BL(X, K)$ is
(a) n^2 (b) $n - 1$ (c) $n + 1$ (d) n

(7) Let $F \in BL(X,Y)$. Then
(a) $\|F\| < \|F'\|$ (b) $\|F\| > \|F'\|$ (c) $\|F\| = \|F'\|$ (d) none of these

(8) In which of the following nlspaces, weak convergence and norm convergence are same?
(a) l^1 (b) l^2 (c) l^p, $p > 2$ (d) none of these

2. Attempt any SEVEN: [14]

(a) Prove: If $E \subset X$ is convex, then E^0 is convex.
(b) Give an example of a linear continuous map on $\mathbb{R}$.
(c) State Hahn-Banach separation theorem.
(d) Let X be a Banach space. If a series $\sum_n x_n$ of elements of X is absolutely summable, then show that it is summable in X.
(e) Prove: If $F: X \rightarrow Y$ is continuous, then F is a closed map.
(f) State open mapping theorem.
(g) Define $\sigma_e(A)$ and $\sigma_a(A)$.
(h) Prove: If $F, G \in BL(X, Y)$, then $(F + G)' = F' + G'$.
(i) Show that if $\{x_n\}$ is a sequence in X and if $x_n \xrightarrow{w} x$ and $x_n \xrightarrow{w} y$, then $x = y$.

(P.T.O.)

3. (a) State and prove Holder's inequality. [6]

(b) For $f \in C[0,1]$, show that $\|f\|_\infty = \sup\{|f(t)| : t \in [0,1]\}$ defines a norm on $C[0,1]$. [6]

OR

(b) Let $F \in BL(X,Y)$. Define a map $\tilde{F} : X/Z(F) \rightarrow Y$ by $\tilde{F}(x + Z(F)) = F(x)$. Show that $\tilde{F}$ is linear and continuous. [6]

4. (a) State and prove Hahn-Banach extension theorem. [6]

(b) Prove: If Y is a Banach space, then $BL(X, Y)$ is complete. [6]

OR

(b) Prove: A subset $E \subset X$ is bounded if and only if f(E) is bounded in K, for every $f \in X'$. [6]

5. (a) Let $F: X \rightarrow Y$ be linear map. Prove that if there exists $\gamma > 0$ such that for every $y \in Y$, there is $x \in X$ with $F(x) = y$ and $\|x\| \leq \gamma\|y\|$, then F is an open map. [6]

(b) Let $A \in BL(X)$ be of finite rank. Show that $\sigma_e(A) = \sigma(A)$. [6]

OR

(b) Let X be a nls and $A \in BL(X)$. Prove that A is invertible if and only if A is bounded below and surjective. [6]

6. (a) Prove: If X′ is separable, then X is separable. [6]

(b) Define weak* convergence in X′. Show that if $\{x'_n\}$ is a sequence in X′ and $x' \in X'$, then $x'_n \xrightarrow{\|.\|} x' \Rightarrow x'_n \xrightarrow{w} x' \Rightarrow x'_n \xrightarrow{w*} x'$. [6]

OR

(b) Let X be a separable nls. Prove that every bounded sequence in X′ has a weak* convergent subsequence. [6]

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(2)