[63] SEAT No._____ No. of Printed Pages : 3

Sardar Patel University

M.Sc. (Mathematics) (Semester-III); Examination 2019

PS03CMTH22: Mathematical Methods-I

Date: 22nd March, 2019 Full Marks: 70

Friday Time: 2:00 pm to 5:00 pm

Instructions:

1. **Attempt all questions.**
2. **Assume usual/standard notations wherever applicable.**
3. **Figures to the right indicate full marks.**

Q- 1 Choose the most appropriate option for each of following question: **[8]**

(1) The inverse Laplace transform of $\frac{1}{s^2+k^2}$

(a) $\sin kt$ (b) $\cos kt$ (c) $K\cos kt$ (d) none of these

(2) The inverse Laplace transform of $\frac{1}{s}$

(a) $1-\sin t$ (b) $1+\cos t$ (c) 1 (d) $1+\sin t$

(3) Find $L\{0\}=$ _____

(a) 1 (b) 0 (c) πs (d) $\frac{\pi}{2}s$

(4) $\sum_{n=1}^{\infty}\frac{1}{n}=$ _____

(a) ∞ (b) $\frac{2}{\pi}$ (c) π (d) none of these

(5) $Z\left\{\frac{x^n}{n!}\right\}=$ _____

(a) $\exp\left(\frac{x}{z}\right)$ (b) exp (x) (c) exp(z) (d) $\exp\left(\frac{z}{x}\right)$

(6) Fourier coefficient a_0 of the Fourier series of 2π periodic function $f(x)=x,\ -\pi<x\le\pi$ is _____

(a) 2 (b) 0 (c) 1 (d) 0.5

(7) $Z\{1^n\}=$ _____

(a) $\frac{1}{z+1}$ (b) $\frac{z}{z+1}$ (c) $\frac{z}{z-1}$ (d) $\frac{1}{z-1}$

(8) For $a \neq 0$ then Fourier transform of $f(ax)$ is

(a) $F(s-a)$ (b) $\frac{1}{a}F\left(\frac{a}{s}\right)$ (c) $e^{-isa}F(s)$ (d) $\frac{1}{a}F\left(\frac{s}{a}\right)$

Q- 2 Attempt **any Seven** **[14]**

(a) Find $L^{-1}\left\{\frac{1}{s^2+5s+6}\right\}$

(b) State Dirichlet theorem for the convergence of Fourier series.

(c) Define Fourier series with Fourier coefficients

(d) Find $L\{1*1\}$

(e) Prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty L\{f(u)\}du$

(f) State Linearity property and 1st shifting theorem for Laplace transform

(g) Find the inverse Z transform of $\cos\alpha n; \quad \alpha \in R$

(h) Solve the Initial value problem for the difference equation
$f(n+1)-f(n)=1, \quad f(0)=0$

(i) Compute the period of $\cos\left(\frac{x}{3}\right)$.

Q-3 (a) Find the Fourier series of a periodic function $f(x)=|x|; \quad -\pi \leq x \leq \pi$ **[6]**

hence find $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$

(b) Compute the Fourier series of the function $f(x)=x-x^2 \; ; \quad -1 \leq x \leq 1$ defined in

hence find $\sum_{n=1}^{\infty}\frac{1}{n^2}$ **[6]**

OR

(b) Compute the Fourier series of a 2π periodic function $f(x)=x^2$ hence find $\sum_{n=1}^{\infty}\frac{1}{n^4}$

Q-4 (a) Compute the Laplace transforms of $\frac{1-\cos 2t}{t}$ and $\frac{\cos 2t-\cos 3t}{t}$ **[6]**

(b) Using Laplace transform Solve $y''+9y=\cos 2t \; ; \quad y(0)=1=y'(0)$ **[6]**

OR

(b) State Convolution theorem and hence obtain Inverse Laplace transform of $\frac{1}{(s-1)(s^2+1)}$

Q-5 (a) Let $a > 0$ then compute Fourier transform of $\exp(-ax^2)$ **[6]**

(b) Define Fourier cosine transform and hence evaluate: $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$ **[6]**

OR

(b) Solve $u_t = ku_{xx}$ $(x, t > 0)$ subject to $u(x,0) = 0$ and $u_x(0,t) = -a$ also both $u, u_x \to 0$ as $x \to \infty$

Q-6 (a) Define Z-transform and its Convolution. State and prove Convolution theorem for Z transform and hence compute $Z^{-1}\left[\frac{z^2}{(z-2)(z-3)}\right]$ **[6]**

(b) State and prove Initial value theorem for Z-transform. **[6]**

OR

(b) Define Z transform. Using Z transform, find 100^{th} term of Fibonacci sequence.