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SEAT No. _____

Sardar Patel University

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SC

M.Sc. (Mathematics) (Semester-III); Examination 2019

PS03CMTH02: Mathematical Methods-I

Date: 22nd March, 2019

Full Marks: 70

Friday

Time: 2:00 pm to 5:00 pm

Instructions:

1. Attempt all questions.
2. Assume usual/standard notations wherever applicable.
3. Figures to the right indicate full marks.

Q-1 Choose the most appropriate option for each of following question: [8]

(1) The inverse Laplace transform of $\frac{1}{s^2 + k^2}$
(a) $\sin kt$ (b) $\cos kt$ (c) $K \cos kt$ (d) none of these

(2) The inverse Laplace transform of $\frac{1}{s}$
(a) $1 - \sin t$ (b) $1 + \cos t$ (c) 1 (d) $1 + \sin t$

(3) Find $L\{0\} =$ _____
(a) 1 (b) 0 (c) πs (d) $\frac{\pi}{2} s$

(4) $\sum_{n=1}^{\infty} \frac{1}{n} =$ _____
(a) ∞ (b) $\frac{2}{\pi}$ (c) π (d) none of these

(5) $Z\left\{\frac{x^n}{n!}\right\} =$ _____
(a) $\exp\left(\frac{x}{z}\right)$ (b) $\exp(x)$ (c) $\exp(z)$ (d) $\exp\left(\frac{z}{x}\right)$

(6) Fourier coefficient a_0 of the Fourier series of 2π periodic function $f(x) = x, -\pi < x \leq \pi$ is _____
(a) 2 (b) 0 (c) 1 (d) 0.5

(7) $Z\{1^n\} =$ _____
(a) $\frac{1}{z+1}$ (b) $\frac{z}{z+1}$ (c) $\frac{z}{z-1}$ (d) $\frac{1}{z-1}$

(8) For $a \neq 0$ then Fourier transform of $f(ax)$ is

(a) $F(s-a)$ (b) $\frac{1}{a}F\left(\frac{a}{s}\right)$ (c) $e^{-isa}F(s)$ (d) $\frac{1}{a}F\left(\frac{s}{a}\right)$

Q-2 Attempt any Seven

[14]

(a) Find $L^{-1}\left\{\frac{1}{s^2 + 5s + 6}\right\}$

(b) State Dirichlet theorem for the convergence of Fourier series.

(c) Define Fourier series with Fourier coefficients

(d) Find $L\{1 * 1\}$

(e) Prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty L\{f(u)\}du$

(f) State Linearity property and 1st shifting theorem for Laplace transform

(g) Find the inverse Z transform of $\cos \alpha n$; $\alpha \in R$

(h) Find $H_2(x)$

(i) Compute the period of $\cos\left(\frac{x}{3}\right)$.

Q-3 (a) Find the Fourier series of a periodic function $f(x) = |x|$; $-\pi \leq x \leq \pi$

[6]

hence find $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

(b) Compute the Fourier series of the function $f(x) = x - x^2$; $-1 \leq x \leq 1$ defined in

hence find $\sum_{n=1}^{\infty} \frac{1}{n^2}$

[6]

OR

(b) Compute the Fourier series of a 2π periodic function $f(x) = x^2$ hence find $\sum_{n=1}^{\infty} \frac{1}{n^4}$

Q-4 (a) Compute the Laplace transforms of $\frac{1 - \cos 2t}{t}$ and $\frac{\cos 2t - \cos 3t}{t}$

[6]

(b) Using Laplace transform Solve $y'' + 9y = \cos 2t$; $y(0) = 1 = y'(0)$

[6]

OR

(b) State Convolution theorem and hence obtain Inverse Laplace transform of $\frac{1}{(s-1)(s^2+1)}$

Q-5 (a) Let $a > 0$ then compute Fourier transform of $\exp(-ax^2)$ [6]

(b) Define Fourier cosine transform and hence evaluate: $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ [6]

OR

(b) Solve $u_t = ku_{xx}$ ($x, t > 0$) subject to $u(x, 0) = 0$ and $u_x(0, t) = -a$ also both $u, u_x \rightarrow 0$ as $x \rightarrow \infty$

Q-6 (a) Orthonormalize $\{1, x, x^2, x^3\}$ over $[-1, 1]$ [6]

(b) Define Hermite polynomials. State its recurrence relation and orthogonal property. [6]

OR

(b) Define Z transform. State and prove Linearity property of Z-transform and

Evaluate: $Z\{(n+1)^2\}$

~~X~~
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