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SEAT No.________

[40]

No. of printed pages: 2

SARDAR PATEL UNIVERSITY
M. Sc. (Semester III) Examination

Date: 21-4-2018 **Time: 2.00 To 5.00 p.m.**

Subject: MATHEMATICS **Paper No. PS03EMTH23 – (Graph Theory – II)**

Total Marks: 70

1. Choose the correct option for each question: [8]

(1) The graph K_9 can be decomposed into copies of
(a) $K_{1,5}$ (b) P_5 (c) P_4 (d) none of these

(2) If all the digits in the Pruffer code are same, then the graph is
(a) Cycle graph (b) Path graph (c) Star graph (d) $K_{n,n}$ $(n > 1)$

(3) The number of spanning trees in K_6 is
(a) 6 (b) 6^2 (c) 6^3 (d) 6^4

(4) If f is a flow on a network N = (V, A), then f(V,{t}) is
(a) f({s},V) (b) f(V,{s}) (c) f({t}, V) (d) none of these

(5) Let A be a matrix with spectrum {–3, –1, 2, 3, 1}. Then det(A) =
(a) –18 (b) 18 (c) 2 (d) 6

(6) Let $G = K_{4,3}$. Then the non-zero eigen values for G is
(a) 6 (b) 4 (c) 2 (d) 3

(7) The Ramsey number R(3, 3) is
(a) 3 (b) < 6 (c) > 6 (d) 6

(8) If E = {1, 2, 3} with base $B_M = \{\{1,2\}, \{1,3\}\}$, then $C_M =$
(a) {{2,3}} (b) {{2}} (c) {{1}, {2,3}} (d) {{2}, {1,2,3}}

2. Attempt any SEVEN: [14]

(a) Define graceful labelling and give one graceful labelling of P_7.

(b) Find Pruffer code of all possible trees for which the degree sequence is (1, 2, 1, 4, 1, 1).

(c) Prove or disprove: In a weighted connected graph, there is a unique shortest spanning tree.

(d) Define u-v vertex separating set and give one example of it.

(e) Let A be a matrix with spectrum {2, 1, –1, –3}. Then find spectrum of A^2.

(f) Prove: If G is k regular graph, then k is an eigen value of G.

(g) Prove or disprove: R(4, 3) = 8.

(h) State Pigeonhole Principle.

(i) Prove: For $X \subset E$ and $e \in E$, $r(X + e) \leq r(X) + 1$.

(P. T. O.)

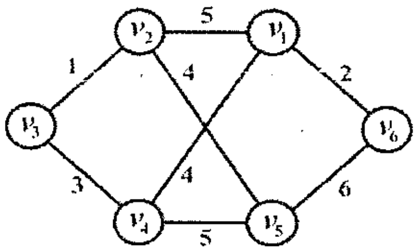
3. (a) Prove: For a set $S \subseteq N$ of size n there are n^{n-2} trees with vertex set S. [6]

(b) Find $\tau(G)$ using Matrix-Tree theorem, for $G = C_5$. [6]

OR

(b) Prove: If $e \in E(G)$ is not a loop, then $\tau(G) = \tau(G - e) + \tau(G \bullet e)$. [6]

4. (a) Using Kruskal's algorithm, find a shortest spanning tree for the graph (A): [6]



(A)

(b) Define a flow, increment of a walk W in a network and give one example of it in a network. [6]

OR

(b) Let f be a flow in a network N = (V, A) with value d. Prove that, if $A(X, \overline{X})$ is a cut in N, then $d = f(X, \overline{X}) - f(\overline{X}, X)$. [6]

5. (a) Prove: For any graph G, $\chi(G) \leq 1 + \lambda_{max}(G)$. [6]

(b) (i) Prove: If G is a bipartite graph, then non-zero eigen values of G occur in pair $(\lambda, -\lambda)$. [6]

(ii) Find $sp(K_{1,3})$.

OR

(b) (i) Prove: For any graph G, $\lambda_{max}(G) \leq \Delta(G)$. [6]

(ii) Prove: If J is a linear combination of powers of A(G), then G is regular.

6. (a) Prove: $R(p, q) \leq R(p-1, q) + R(p, q-1), \forall\ p, q > 2$. [6]

(b) Prove (ANY ONE): In a hereditary system, [6]

(i) Uniformity property (U) $\Rightarrow$ Sub modularity property (R).

(ii) Augmentation property (I) $\Rightarrow$ Uniformity property (U).

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