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SARDAR PATEL UNIVERSITY

M.Sc. (Semester - III) Examination (NC)
Thursday April 19, 2018
Time: 02:00 p.m. to 05:00 p.m.
Subject: Mathematics
Course No. PS03EMTH13 (Operations Research)   Total Marks: 70

Note: (1) All the questions are to be answered in the answer book only.
      (2) Figures to the right indicate marks of the respective question.

Q-1 Choose the most appropriate answer from the options given.   (08)

1. _____ method cannot be used to solve an LPP with more than 4 decision variables.
   (a) Graphical   (b) Big M (c) Simplex   (d) Two-phase

2. All the constraints of an LPP when expressed in the standard form are of _____ type.
   (a) $\leq$ (b) $\geq$ (c) $\neq$ (d) $=$

3. In graphical method, the feasible region is always _____.
   (a) a triangle   (b) open   (c) convex   (d) an empty set

4. A basic feasible solution is called _____ if all the basic variables are non-zero.
   (a) non-basic   (b) infeasible   (c) non-degenerate   (d) degenerate

5. An assignment problem can be solved by _____ method.
   (a) Simplex   (b) Hungarian   (c) Big M (d) Graphical

6. In a transportation problem if the supply exceeds the demand, then _____.
   (a) dummy source is added   (c) a demand is reduced
   (b) a source is reduced   (d) dummy destination is added

7. In an assignment problem the number of tasks _____ number of men.
   (a) $<$ (b) $>$ (c) $=$ (d) none of these

8. If one of the constraints is non-linear, then the problem is _____.
   (a) LPP   (b) NLPP   (c) Partial LPP   (d) Partial NLPP

Q-2 Attempt *Any Seven* of the following: (14)

(a) What is the use of a slack variable in LPP?

(b) What is meant by an initial basic feasible solution?

(c) Express the following LPP in the standard form:
    Maximize $z = 3x_1 - 5x_2$ subject to $x_1 + 2x_2 \leq 3$, $2x_1 + x_2 \leq 4$, and $x_1, x_2 \geq 0$.

(d) In an LPP, show that the set of feasible solutions is a convex set.

(e) In an LPP how an equality constraint is dealt with?

(f) What are the constraints in an assignment problem?

(g) Explain the term "dummy source" in a transportation problem.

(h) What is the use of uv-method?

(i) In an NLPP how Lagrangian is defined?

Q-3 (a) State and prove the fundamental theorem of linear programming.   (06)

(b) A farmer has a supply of chemical fertilizer of Type-I which contains 10% Nitrogen,   (06)
    and 6% of Phosphoric acid and Type-II fertilizer which contains 5% Nitrogen and 10%
    Phosphoric acid. After testing the soil conditions of a field it is found that at least 14 kg
    of Nitrogen and 14 kg of Phosphoric acid is required for a good crop. The fertilizer of
    Type-I costs ₹ 2/kg and that of Type-II costs ₹ 3/kg. Using graphical method, determine
    how many kilograms of each fertilizer should be used such that the cost is minimum?

(P.T.O.)

OR

(b) Solve the following LPP by the simplex method:

Max $z = 4x_1 + 10x_2$

subject to $2x_1 + x_2 \leq 50$, $2x_1 + 3x_2 \leq 90$, $2x_1 + 5x_2 \leq 100$, and $x_1, x_2 \geq 0$. (06)

Q-4 (a) Describe the Two-phase method for solving an LPP. (06)

(b) Solve the following LPP by Big M method: (06)

Max $z = 6x_1 + 4x_2$

subject to $2x_1 + 3x_2 \leq 30$, $3x_1 + 2x_2 \leq 24$, $x_1 + x_2 \geq 3$, and $x_1, x_2 \geq 0$.

OR

(b) Obtain the dual of the following problem: (06)

Max $z = 5x_1 + 6x_2$

subject to $x_1 + 2x_2 = 5$, $-x_1 + 5x_2 \geq 3$, $4x_1 + 7x_2 \leq 9$, and x_1 unrestricted, $x_2 \geq 0$.

Q-5 (a) State various methods to obtain initial basic feasible solution to a transportation problem and describe any one of them. (06)

(b) Obtain the initial basic feasible solution of the following transportation problem by Northwest-corner method. (06)

	D	E	F	G	Availability
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

OR

(b) Solve the following assignment problem by Hungarian method. (06)

Men	Jobs			
	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

Q-6 (a) Obtain the bordered Hessian matrix for the objective function (06)

$z = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$, the constraints are $x_1 + x_2 \leq 1$, $2x_1 + 3x_2 \leq 6$ and $x_1, x_2 \geq 0$.

(b) What is meant by bordered Hessian matrix? Explain its use. (06)

OR

(b) Obtain the conditions for Max $z = x_1^2 + 3x_2^2 + 5x_3^3$ subject to (06)

$x_1 + 2x_2 + 3x_3 = 2$, $x_1 + 2x_2 + x_3 = 5$, and $x_1, x_2, x_3 \geq 0$.

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