SC

SEAT No.________

[23]

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M.Sc. (Sem-III), PS03EMTH11, Mathematical Probability Theory;
Tuesday, 24^{th} April, 2018; 02.00 p.m. to 05.00 p.m.

Maximum Marks: 70

Note: (i) Notations and terminologies are standard; (ii) Figures to the right indicate marks.

Q.1 Answer the following. [8]

1. Let $(\Omega, \mathcal{A}, P)$ be probability space and $\{A_i\} \subset \mathcal{A}, A = \bigcup_{i=1}^{\infty} A_i$. Then $P(A)$ is
   (A) $< \sum_{i=1}^{\infty} A_i$ (B) $\leq \sum_{i=1}^{\infty} A_i$ (C) $\geq \sum_{i=1}^{\infty} A_i$ (D) $= \sum_{i=1}^{\infty} A_i$
2. Let F be a distribution function of r.v. X. Then for any $a, b \in \mathbb{R}$ with $a > b$, $P(b < X \leq a)$ is
   (A) $F(b) - F(a)$ (B) $F(a) - F(b)$ (C) $a - b$ (D) $b - a$
3. Which is not true from the following ?
   (A) convergence in probability $\Rightarrow$ convergence in distribution
   (B) almost sure convergence $\Rightarrow$ convergence in probability
   (C) convergence in distribution $\Rightarrow$ convergence in probability
   (D) at least one of (A),(B),(C) is true
4. If $X_n \xrightarrow{L} X$, then F_X
   (A) is differentiable (B) is continuous but not differentiable
   (C) need not be continuous (D) none of (A),(B),(C) is true
5. If $\phi(u)$ is characteristic function of random variable X, then
   (A) $\phi(-u) > \phi(0)$ (B) $\phi(-u) > \overline{\phi}(u)$
   (C) $\phi(-u) = \overline{\phi}(u)$ (D) none of (A),(B),(C) is true
6. If $\varphi(u)$ is characteristic function of random variable X, then the characteristic function of $1 + 2X$ is
   (A) $e^{2iu}\varphi(u)$ (B) $e^{-2iu}\varphi(u)$ (C) $e^{iu}\varphi(2u)$ (D) $e^{-iu}\varphi(2u)$
7. The pdf of standard normal random variable is
   (A) odd function (B) even function
   (C) neither even nor odd (D) none of these
8. Let F be a distribution function and h be corresponding characteristic function. For any $u > 0$, $\exists K > 0 \ni \int_0^u [h(0) - Re(h(v))]dv$
   (A) $\geq \frac{K}{u} \int_{|x| \geq \frac{1}{u}} dF(x)$ (B) $\geq \frac{u}{K} \int_{|x| \geq \frac{1}{u}} dF(x)$
   (C) $< \frac{u}{K} \int_{|x| \geq \frac{1}{u}} dF(x)$ (D) none of these

Q.2 Attempt any *seven*: [14]

(a) Define probability measure.
(b) If $P(A) = 0.25$ and $P(B) = 0.8$ then show that $0.25 \geq P(A \cap B) \geq 0.05$.
(c) Let $\{X_n\}$ be a sequence of random variables with $E(X_n) = 2$ and $\text{Var}(X_n) = \frac{1}{n}$, $\forall n$. Does X_n converge in probability ? Justify.
(d) Define distribution function.
(e) Define weak convergence.
(f) What is characteristic function of constant r.v.?
(g) State Inversion theorem for characteristic function.
(h) State Weak Law of Large Numbers.
(i) State Levy's theorem.

[P.T.O.]

Q.3

(a) Show that the distribution function, F_X of r.v. X is non-decreasing, right continuous, $F_X(\infty) = 1$ and $F_X(-\infty) = 0$. [6]

(b) Let $\Omega = \{HH, HT, TH, TT\}, \mathcal{A} = \{\phi, \Omega, TT, HH, \{TT, HH\}, \{HT, TH\}, \{HH, HT, TH\}, \{TT, HT, TH\}\}$ and $X : \Omega \to \mathbb{R}$ defined as the number of H appears. Is X a r.v ? [6]

OR

(b) Show that $X_n \xrightarrow{P} X$ if $X_n \xrightarrow{a.s} X$. Does the converse true ? Justify.

Q.4

(a) State and prove Jordan Decomposition Theorem. [6]

(b) If $X_n \xrightarrow{P} X$ then show that $X_n \xrightarrow{L} X$. [6]

OR

(b) Prove or disprove : $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{L} Y \Rightarrow X_n + Y_n \xrightarrow{L} X + Y$.

Q.5

(a) Prove that every characteristic function is continuous on $\mathbb{R}$ and $|\phi(u)| \leq \phi(0)$. [6]

(b) State and prove weak compactness theorem. [6]

OR

(b) Let f be a pdf of r.v.X and f is even function. Then show that the characteristic function of X is real valued.

Q.6

(a) State Chebychev's inequality and hence prove Weak Law of Large Numbers. [6]

(b) State and prove Central Limit Theorem. [6]

OR

(b) State and prove Kolmogorav's inequality.

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