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SARDAR PATEL UNIVERSITY

M.Sc. (Mathematics) Semester - III Examination

Monday, 16th April, 2018

PS03EMTH02, Banach Algebras

Time: 02:00 p.m. to 05:00 p.m. Maximum marks: 70

Note: Assume standard notations wherever applicable. Unless otherwise mentioned $\mathcal{A}$ is a unital Banach algebra.

Q-1 Write the question number and appropriate **option number only** for each question. [8]

(a) In _____, the spectrum of an element need not be compact.
(i) $\mathcal{P}[0,1]$ (ii) $C[0,1]$ (iii) ℓ^∞ (iv) $A(\mathbb{D})$

(b) $f \in C[0,1]$ defined by $f(x) =$ _____, $(x \in [0,1])$, is invertible.
(i) x (ii) x^2 (iii) x^3 (iv) $\cos x$

(c) $\varphi : A(\mathbb{D}) \to \mathbb{C}$ defined by $\varphi(f) = f^*(i)$, $(f \in A(\mathbb{D}))$ is _____.
(i) multiplicative and linear (iii) multiplicative but not linear
(ii) linear but not multiplicative (iv) neither multiplicative nor linear

(d) Boundary of G is _____.
(i) open (ii) nonempty (iii) disconnected (iv) finite

(e) Gel'fand space of $C[2,3]$ is homeomorphic to _____.
(i) $(2,3)$ (ii) $[0,1]$ (iii) $\mathbb{R}$ (iv) $\mathbb{N}$

(f) Spectral radius is isometry for _____.
(i) $C[0,1]$ (ii) $C^1[0,1]$ (iii) $M_2(\mathbb{C})$ (iv) ℓ^1

(g) _____ is a C^*-algebra.
(i) ℓ^1 (ii) $A(\mathbb{D})$ (iii) $C^1[0,1]$ (iv) $\mathbb{C}$

(h) Spectrum of _____ element of a C^*-algebra is real.
(i) selfadjoint (ii) normal (iii) invertible (iv) unitary

Q-2 Attempt *Any Seven* of the following: [14]

(a) Define and give an example of a normed algebra.

(b) Give an example of two elements of a Banach algebra such that $\|xy\| < \|x\|\|y\|$. Verify your claim.

(c) For the identity e of $\mathcal{A}$, prove that $\|e\| \geq 1$.

(d) Prove that multiplication on a Banach algebra is jointly continuous.

(e) Find the spectrum of $(1,2,3) \in \mathbb{C}^3$.

(f) Define the *radical* of a Banach algebra.

(g) Define and give an example of a complex homomorphism of a Banach algebra.

(h) Show that $\mathbb{C} \times \{0\}$ is a maximal ideal of $\mathbb{C}^2$.

(i) Define involution on $A(\mathbb{D})$ and show that $g(z) = z$, $(z \in \mathbb{D})$ is selfadjoint.

[P.T.O]

Q-3 (j) Show that $C[0,1]$ is complete with the supnorm. [6]

(k) Show that the map $x \in G \mapsto x^{-1} \in G$ is a homeomorphism. [6]

OR

(k) Show that boundary of S is subset of Z. [6]

Q-4 (l) For $x \in \mathcal{A}$, define *spectrum of* x, obtain resolvent equation and using that show that $\text{sp}_{\mathcal{A}}(x)$ is nonempty. [6]

(m) Show that complex homomorphism on a Banach algebra is continuous. [6]

OR

(m) Let $\mathcal{A}$ be a unital Banach Algebra and $x \in \mathcal{A}$. Show that $r(x) = \lim_{n\to\infty} \|x^n\|^{\frac{1}{n}}$. [6]

Q-5 (n) Show that the set of all nonzero complex homomorphisms on $\mathcal{A}$ and the set of all maximal ideals of $\mathcal{A}$ are in one one correspondence. [6]

(o) Characterize closed ideals of $C[0,1]$. [6]

OR

(o) Describe the topology of the Gel'fand space of a commutative unital Banach algebra. [6]

Q-6 (p) Define C^*-algebra and for a compact T_2-space X, show that $C(X)$ is a C^*-algebra. [6]

(q) State and prove Banach-Stone's theorem. [6]

OR

(q) For a normal element x of a C^*-algebra, sow that $\|x^2\| = \|x\|^2$. [6]

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